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Integration of the inverse dynamics with a reference model technique, and its application for the improvement of the helicopter flying qualities

Z Mbikayi*, A V Efremov and E V Efremov

Flight Dynamics and Control Department, *Moscow Aviation Institute* (National Research University), 4 Volokolamskoe shosse, Moscow125993, Russia

*E-mail: pvl@mai.ru

Abstract. The application of inverse dynamics for the improvement of flying qualities is considered. It is shown that the use of the inverse dynamics in the control loop suppresses the dynamics of the aircraft considerably, leaving only the dynamics of the integral in almost the entire frequency range. The inverse dynamics however could not help in case of atmospheric disturbances or modeling uncertainties. A new scheme using a reference model and its inverse was presented to counter these effects. The results of the mathematical modeling showed that the inverse dynamics allow to improve the tracking performances and all other parameters of the pilot-aircraft system compared to the controller synthesized with feedback gains only. These results were confirmed using ground based simulation. These simulations showed that the use of the inverse dynamics based technique allowed to decrease the variance of error up to 60% during a tracking compensation task. The simulations also showed to that the addition of the reference model and its inverse proved to be able to reduce or suppress atmospheric turbulence, couplings between different control channels and effects of modeling uncertainties, while improving the tracking accuracy and decreasing the pilot workload, thus enhancing the flying qualities of the helicopter.

1. Introduction

One of the major areas in flight control is the provision of necessary flying qualities. Several techniques developed recently allow to change the dynamics of the aircraft considerably. One of them is the dynamic inversion approach introduced in [1].It is a nonlinear control technique based on feedback linearization. This technique has been used successfully over the years for in a variety of application for both airplanes [2] and rotorcrafts [3]. It is used to make an appropriate coordinate transformation of the nonlinear plant, so that any linear control method can be used on the resulting linear plant. In [3] and [4] for example, the non linear dynamic inversion is used in the inner loop, and the outer loop consists of a PID based controller. And in [5], the robustness is provided by an H-infinity based controller. As a linear control technique, it has been used as an addition in the feedforward loop to support the feedback controller. And in this form, its integration with reference models techniques has been studied by [6] and others. In these studies, the desired dynamics from the pilot stick commands are computed via the use of reference models, which are then given to the inverse dynamics in order to compute the actuators positions necessary to achieve those desired dynamics. This method however still requires an exact model of the aircraft to be controlled. Therefore, a PID - type compensator must be added to provide the necessary robustness for modeling uncertainties and disturbances as in [3].

This paper essentially considers a different use of the dynamic inversion technique as a linear control method. A novel scheme using the inverse dynamics, reference models and their inverses is presented in order to enhance the robustness of the controller and provide better tracking performances. The resulting controller that can be used either on a linear plant or on a plant where the feedback linearization based dynamic inversion has been used in the inner loop.

The efficiency of the proposed technique is evaluated by comparing it to the use of feedback gains-based controllers and controllers synthesized with the inverse dynamics and feedback gains that provide robustness.

2. Using the inverse dynamics

The inverse dynamics can be used in the feedforward loop to improve tracking performances. As stated in [7], the inverse dynamics requires a stable system. For the vehicle used, stability can be achieved using a gain of matrix used in full stated feedback, computed using the pole placement method that uses the formula of Ackermann [8] through the controllability matrix [9], in order to compute the stabilization gains.

To achieve zero tracking error for a given reference signal r*(s)* in a controllable and stable system $G(s)$ with a control loop as given in figure 1, the input *u(s)* can be found as:

$$y(s) = r(s) = u(s) \cdot G(s) \tag{1}$$

$$u(s) = \frac{r(s)}{G(s)} = r(s) \cdot G^{-1}(s) \tag{2}$$

Figure 1 shows the use of the inverse dynamics in the control loop, with the stabilizing matrix of gains.

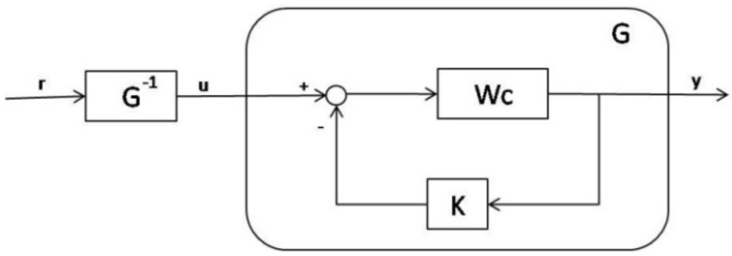


Figure 1. Inverse of closed loop.

Where

$$G = \frac{W_c}{1 + W_c \cdot K}$$

W_c is the aircraft dynamics.

With the pilot in the loop, the goal is to have only one integral between the pilot command and the controlled state. Figure 2 demonstrates the frequency response characteristics of the longitudinal dynamics of the helicopter equipped with the feedback providing the necessary stability, and the same dynamics equipped with the inverse dynamics. It is seen that the inverse dynamics change the controlled element dynamics, approaching it to the integral in practically all the frequency range.

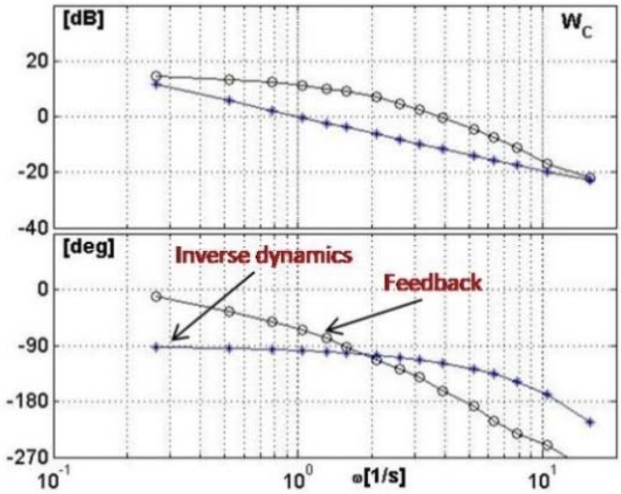


Figure 2. Frequency response inverse dynamics vs. feedback alone.

3. Constructing the Inverse

After making sure that the aircraft is stable, the inverse dynamics can be computed.

For a transfer function $\frac{y(s)}{u(s)}$ the inverse is simply $\frac{u(s)}{y(s)}$.

The stable plant *G* given in figure 1 is a multiple input multiple output (MIMO) system, and can be representedin state space form as:

$$sX(s) = AX(s) + BU(s) \quad (3)$$
$$Y(s) = CX(s) \quad (4)$$

where *A, B* and *C* are the state matrix, input matrix and output matrix respectively; *U, X* and *Y* are the control vector, state vector and output vector respectively; *s* is the Laplace transform variable.

The inverse can be computed analytically as shown in [7].

From (3) we get

$$X(s) = (sI - A)^{-1}BU(s) \quad (5)$$

Replacing (5) in (4) we get $Y(s) = C(sI - A)^{-1}BU(s)$.

From this, we can get the relationship between the transfer function and the state space representation as:

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B \quad (6)$$

Since$A$,B, C and D are matrices, equation (6) is valid only if the number of rows of the matrix C is equal to the number of columns of matrix B. Which means the system needs to be square (Number of inputs = number of outputs), such that each input controls a specific output.

The inputs here are the pilot commands: *Collective, Longitudinal cyclic, Lateral cyclic* and *pedals*. The outputs are: *Climb rate, Pitch rate, Roll rate* and *Yaw rate*.

The inputs and outputs are coupled as follow:

$$\begin{bmatrix} Collective & = & Climb \quad rate \\ Longitudinal & = & Pitch \quad rate \\ Lateral & = & Roll \quad rate \\ Pedals & = & Yaw \quad rate \end{bmatrix} \quad (7)$$

From (6), we get a $(4x4)$ matrix of transfer functions corresponding to couplings of each input and each output as shown in matrix G where G_{ij} represents the transfer function from input i to output$j$.

$$G = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \tag{8}$$

Now each transfer function is inversed, and we get a new $(4x4)$ matrix G^* of inversed transfer functions which is put back together in state space form.

However, since all the transfer functions in G are strictly proper, their inverses are improper. Additionally, in the mathematical model used, the actuator dynamics were modeled as a first order system, and there was a time delay in the system that was approximated as a third order system. It was therefore necessary to use a fifth order low pass filter F, to make the inverse dynamics proper. This filter is used not only to proper the system, but also to filter the high frequencies of the inverse dynamics that might lead to non physical actuator rates. And as shown in the next section, the dynamics of this filter is used as the reference model. The frequency of the filter was chosen by experimenting with different frequencies, by trial and error. And the frequency was chosen such that it is high enough not to change the dynamic of the inverse dynamics, but not to high as to keep the actuators rates within acceptable limits.

4. Reference model

As shown by figure 1, the inverse dynamics are calculated for the closed loop system and are used alone in the feedforward. A feedforward loop cannot achieve robustness in case of plant modeling uncertainties, or disturbances. A new scheme using a reference model and its inverse is introduced to provide robustness, as shown in figure 3. The scheme is extended and used for the MIMO system with four inputs and four outputs. This helps suppress the couplings between the aircraft dynamics as well. The reference model is chosen to be the filter *F* used with the inverse dynamics. This choice was led by the fact that the dynamics of the filter are very close to the dynamics of the entire system. Equation (9) shows that in case of an uncertainty-free system, the dynamics of the entire system will be practically the same as those of the filter used in with the inverse dynamics. Choosing the reference model dynamics to be the same as those of the filer allows it to only act when equation (9) is not true, which means there are modeling uncertainties. Because the inverse dynamics of the filter will also be improper, a second filter *F2* is introduced in order to make the inverse proper. The frequency of this filter also needs to be chosen carefully such it is not too low as to limit the effect of the reference model, and not to high as to let through high frequency signals that might saturate the actuators.

$$G^{-1}(s) \cdot F(s) \cdot G(s) = F(s) \tag{9}$$

The concept of reference model introduced in this paper is simply a method that the controller uses in order to keep suppressing the aircraft dynamics even in case of uncertainty, only leaving an integral-type response as per flying qualities requirements given in [10] which allows the pilot frequency response to be gain-like.

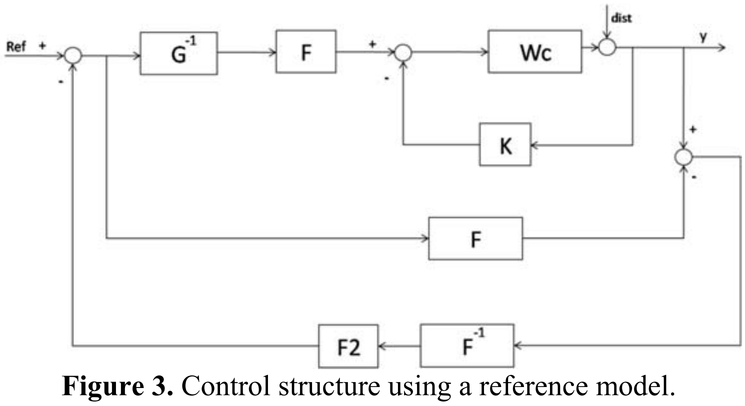


Figure 3. Control structure using a reference model.

5. The investigation of effectiveness of the proposed principle of inverse dynamics

The effectiveness of the proposed approach was tested by mathematical modeling and in ground based simulation. Modeling uncertainties introducing differences between the inverse dynamics and the plant model are simulated by changing the aerodynamic coefficients, stability derivatives and poles of the linear model of the plant. Additionally, time delays and nonlinear actuator dynamics with rate limits are added to the system in simulations.

5.1. Time domain simulations

Simple time domain verification is done by giving a step input in the longitudinal channel, and observing the behavior of the aircraft.

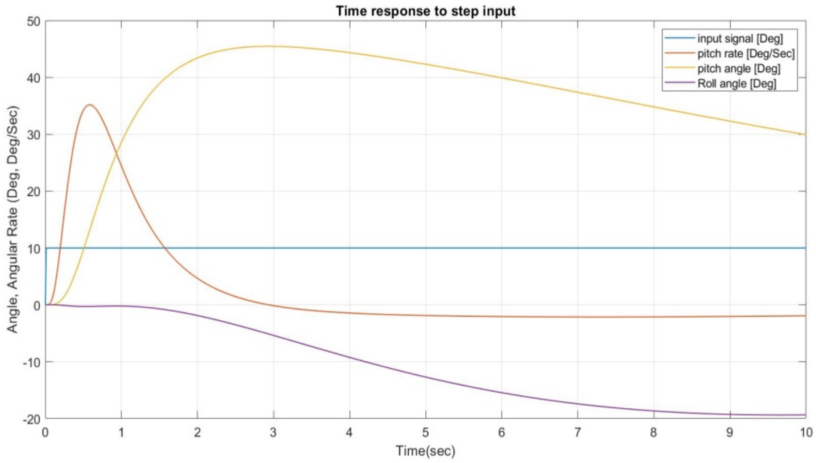


Figure 4. Time response to longitudinal step input (feedback controller).

The time responses to a step input in the longitudinal channel of the helicopter equipped with the feedback controller are show on figure 4. The tracking performances are very bad as neither the pitch rate, nor the pitch angle can track the reference signal. Additionally, the roll angle deviates considerably from the steady state value. This shows strong coupling between the control axes. This will result in a high pilot workload because the pilot needs to be actively controlling the axis in which the task needs to be achieved, and needs to control other axes as well, in order to suppress the coupling effects. The same step input is given to the helicopter equipped with the inverse dynamics and reference model controller, and the time responses are shown on figure 5. The tracking performances are greatly improved and the coupling effects are practically suppressed.

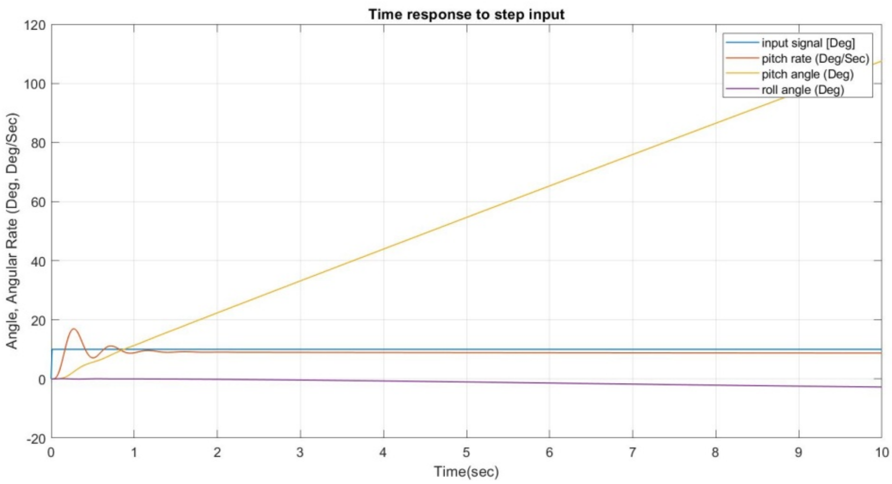


Figure 5. Time response to longitudinal step input (inverse + reference model).

5.2. Mathematical modeling of the pilot-aircraft system with the inverse dynamics

The investigated pilot-aircraft system is shown on figure 6.

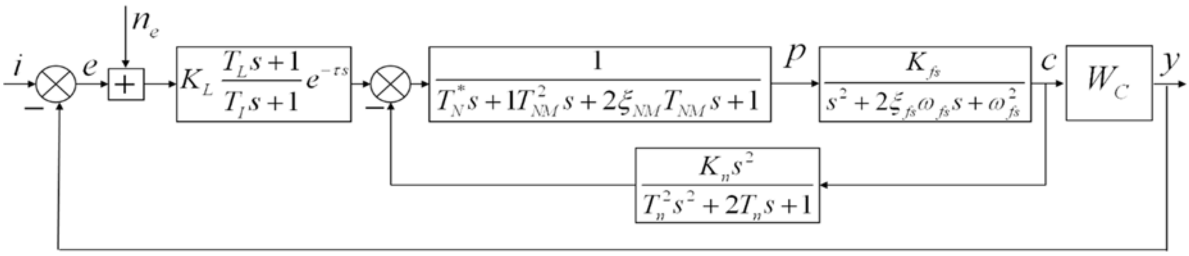


Figure 6. Pilot-aircraft structural model.

For the modeling, the modified pilot-aircraft structural model considered in [11] and [12] was used. This model is based on the well known Hess structural model [13]. The input signal was simulated as a random signal with the spectral density $S_{ii} = \frac{K^2}{(\omega_i{}^2 + 0.5^2)}$ and variance $\sigma^2 = 4\ cm^2$. As for the model of the vehicle, a middle- sized helicopter was used. The frequency response characteristics of its dynamics are shown on figure 2.

The selection of the parameters K_L, T_L, K_n, T_n, of the pilot-aircraft structural model was done according to the parameter optimization procedure considered briefly in [11] and [12]. As for the parameters $\tau, \xi_{fs}, \omega_{fs}$, they are the same as given in [11] and [12] and are equal to 0.2 sec, 0.25 and 14 rad/sec respectively. The results of the mathematical modeling for the helicopter with feedback and helicopter with feedback + inverse dynamics laws are given on figure 7. The variance of error can be seen with its components: The variances of error correlated with the input signal $\sigma_{e_i}{}^2$, and with the pilot's remnant $\sigma_{e_n}{}^2$. Additionally, the frequency response characteristic of the pilot-vehicle closed loop system is shown.

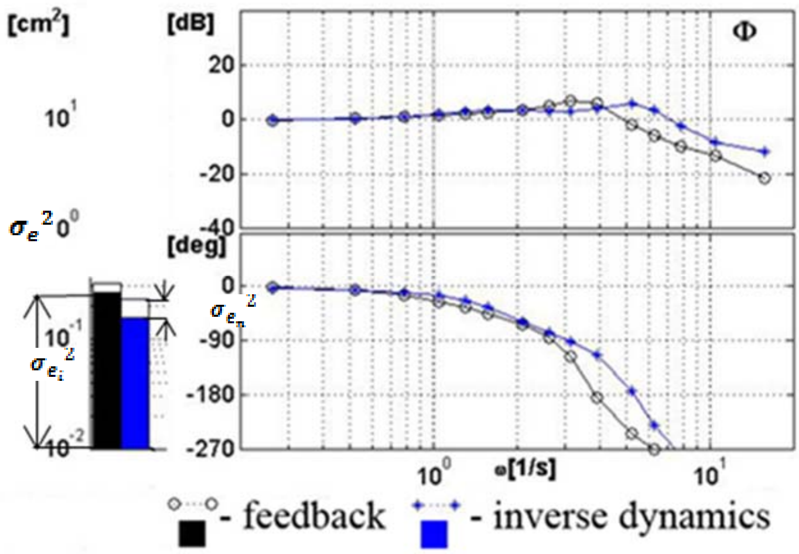


Figure 7. Result of mathematical modeling.

It is seen that the inverse dynamics allow to decrease the variance of error up to 30 %, increase the bandwidth of the closed loop system up to 13 % and decrease the resonant peak up to 12.6 %.

The results of mathematical modeling in case of uncertainties can be seen on figure 8. Uncertainties were simulated by changing stability derivatives of the mathematical model of the helicopter up to 40%. It was also simulated by changing the poles of the system, and the results were the same. It can be seen that it was necessary to add the reference model in order to suppress the effects of uncertainties.

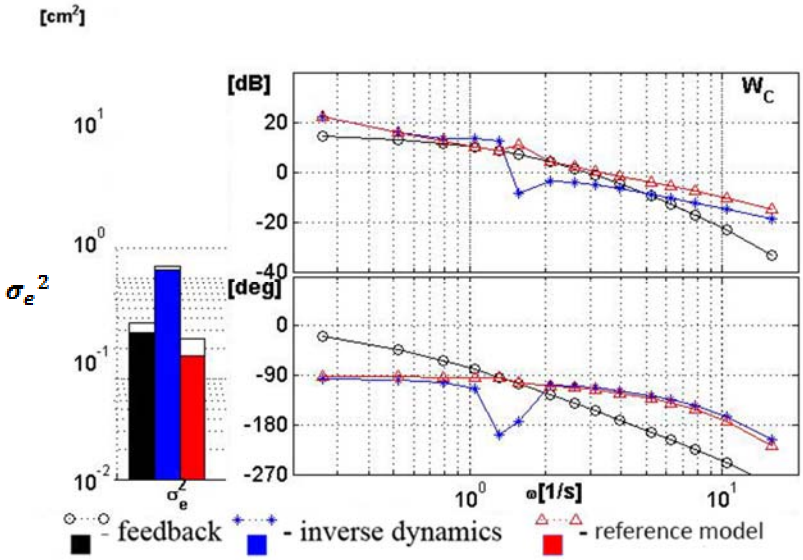


Figure 8. Result of mathematical modeling with modeling uncertainties.

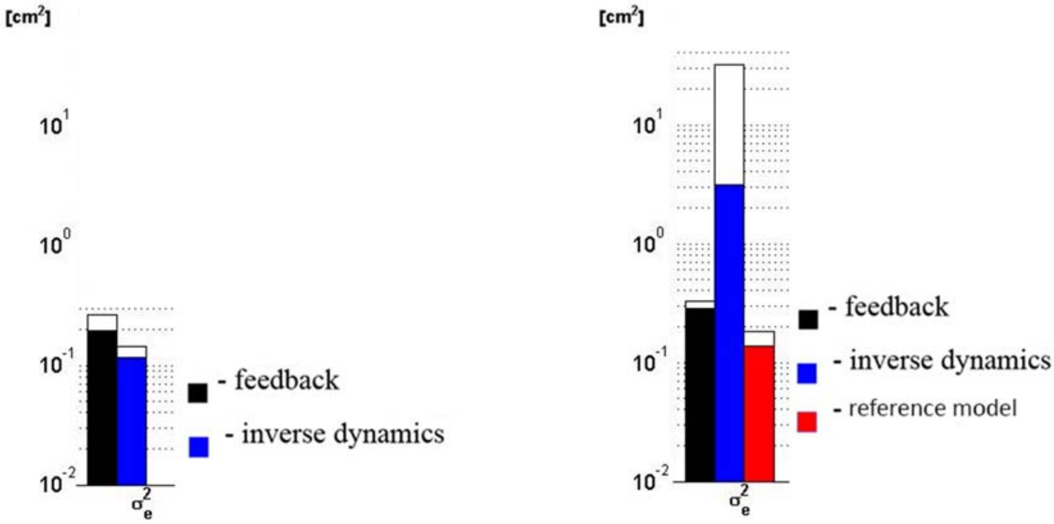
The longitudinal transfer functions of the helicopter are used in the mathematical modeling of the pilot in order the get the results shown above. Additional experiments were carried out with the full linear and coupled model of the helicopter in a ground based simulator, and the results were very close to those obtained above

5.3. Results of ground based simulations

Experiments were carried out on one of the simulators of the pilot vehicle laboratory of MAI. The pilot had to carry out a compensatory tracking task. The experiments were carried out with the same dynamics used in the mathematical modeling. The input signal was the polyharmonic signal $i(t) = \sum_{k=0}^{15} A_k \cos \omega_k t$ where the amplitudes A_k and ω_k were selected according to the technique given in [13].

Figure 9 shows the results of the experiments. In addition to the variance of error which is decreased by up to 80% on figure 9.a, analyses of the pilot performances showed a lower phase lag, which can be translated as less pilot workload during the tracking task.

The variance of error in case of 40 % uncertainties in the knowledge of the aircraft model is shown in figure 9b. It can be seen that when the inverse dynamics are used alone, the error variance is large compared to the use of the feedback alone. It is therefore necessary to use the reference model in order to suppress the uncertainties. It is seen that the use of the reference model allows to decrease the variance of error up to 60 % compared to the feedback alone. The results obtained in ground based simulations showed that the variance of the error decreased even more than it did when running the mathematical modeling simulations.



(a) (b)

Figure 9. Results of experiments: (a) experiment without uncertainties; (b) experiment with modeling uncertainties.

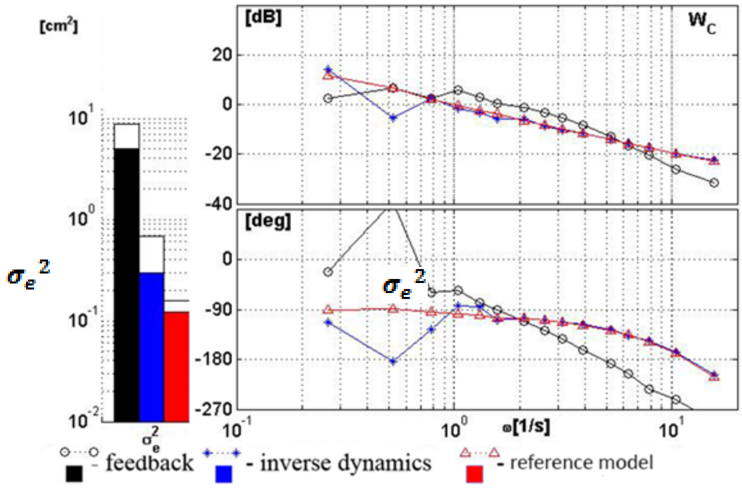


Figure 10. Experiments with atmospheric turbulence.

Figure 10 shows the responses when atmospheric turbulence was added to the simulation (disturbance is added on figure 2 as dist). The turbulence was simulated with a set of harmonics with frequencies non-orthogonal to the input signal. The results showed that the use of the inverse dynamics alone provided good flying qualities when the turbulence had small amplitudes and frequencies.

As the amplitude and the frequency increased, the performances got worse, and eventually, the aircraft was uncontrollable after reaching some threshold, due to periodic instability. From figure 10 it is seen that the turbulence influences considerably on the controlled element dynamics equipped with the feedback only, and the one with the inverse dynamics only. This influence is associated with the couplings between the longitudinal and lateral channels. As for the version equipped with the inverse dynamics and the reference model, it stays practically insensitive to the influence of the turbulence and the variance of error is decreased considerably compared to the feedback alone, or the inverse dynamics alone.

6. Conclusion

A technique was proposed for the integration of the inverse dynamics and reference model. This technique allows to suppress the modeling uncertainties, disturbances, and dynamics couplings. Mathematical modeling demonstrated that the proposed technique allows a considerable improvement of performances and other parameters of the pilot-aircraft system. These results were confirmed with ground based simulations. These simulations demonstrated that the flight control system developed with help of the proposed technique allowed to improve performances and robustness, suppressing the effects of atmospheric turbulence, and decreasing the variance of the error up to 45% when there is no uncertainties, and up to 60% in case of modeling uncertainties. Additionally, it helps decrease the pilot workload considerably, increase the bandwidth of the closed loop system and decrease the resonance peak of the closed loop system, thus, enhancing the flying qualities of the helicopter, placing them in level 1 when measured using the Cooper Harper scale.

Acknowledgments

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