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H-infinity approach to design optimal output control for the aggregated interpolation model of helicopter motion

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Abstract. This paper presents the problem of designing a feedback control of the aggregated model describing helicopter flight dynamics under the influence of exogenous factors based on the H-infinity approach. The original nonlinear equations of dynamics are assumed to be linearized regarding to several flight modes that characterize with a set of precisely specified flight conditions such as an attitude of an aircraft, its forward speed and turn rate. Matrices of the linearized systems are dependent on the speed of the aircraft. Due to that the aggregated model comprises eight submodels corresponding to flight modes when the speed varies from hover to 140 kts in a certain regular interval. The proposed principle of designing includes two stages: on the first stage the feedback control input is selected for each submodel separately so that the H-infinity norm of the value functional with respect to the H-infinity norm of disturbances is minimum; on the second one the interpolation technique lying in the expansion of matrices in the basis of finite functions is applied. The solution method is experienced on three helicopter models: ZD559-Lynx, the twin engine, utility helicopter in the 4.5 -ton category; S123-Bo105, the twin engine helicopter in the 2.5 ton class; SA330-Puma, the twin engine, medium-support helicopter in the 6-ton category.

1. Introduction

The complexity of helicopter flight dynamics makes the problem of a regulator design to stabilize a helicopter system under the effect of disturbances tough to solve [1,8]. Due to importance of tasks performed by helicopters this problem gains a considerable attention from many researches. To provide the good robustness and handling quality of a system especially if uncertainties of different types are needed to take into account the H-infinity strategy is useful [2,4-6]. The H-infinity design problem can be stated as the problem of searching a maximum value of an uncertainty such that the desired regulator provides the stability of a closed-loop system with bounded disturbances in accordance with the H-infinity norm. There exist several implementation of the H-infinity approach among which algorithms where more than one associated Ricatti equations are required to solve or the equivalent LMI strategy can be distinguished [6-7]. In the paper, this approach is implemented as the recursive procedure of solving one Riccati equation without a preliminary initialization of the system gain that produces the desired regulator [2].

2. System model

Consider the dynamical model of helicopter motion containing the state equation that describes the evolution of a state vector $x(t)$ for a given control input $u(t)$ provided that the motion is undergone by exogenous disturbances $d(t)$ of a known intensity like strong wind, wind gusts, low level wind shear and the output equation for an output vector $y(t)$:

$$\begin{aligned} \dot{x}(t) &= A_i\, x(t) + B_i\, u(t) + D_i\, d(t), \quad i = 1,...,m \\ y(t) &= C_i\, x(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^k$, $d(t) \in R^s$, $A_i = A(V_i) \in R^{n\times n}$, $B_i = (V_i) \in R^{n\times k}$, $D_i \in R^{n\times s}$, $C_i \in R^q$. The subscript i indicates a mode of the motion. In the paper one considers 8 modes $(m = 8)$ which correspond to the flight on sea level with zero sideslip and turn rate provided that the helicopter forward speed V_i varies from hover to 140 kts: $V_i = (i-1)\Delta V_i$, $\Delta V_i = 20$ kts, $i = 1,..,8$.

The equations in the system (1) are linearized equations of helicopter flight dynamics in a neighborhood of the aforementioned trim modes [1]. With this regard letting the state vector $x = (u, w, q, \theta, v, p, \varphi, r)^T$ and the control input $u = (\theta_0, \theta_{1s}, \theta_{1c}, \theta_{0T})^T$ the matrices A_i and B_i are represented as

$$A_i = \begin{bmatrix} X_u & X_w & X_q - W_e & -g\cos\Theta_e & X_v + R_v & X_p & 0 & X_r + V_e \\ Z_u + Q_e & Z_w & Z_q - U_e & -g\cos\Phi_e \sin\Theta_e & Z_v - P_e & Z_p - V_e & -g\sin\Phi_e \sin\Theta_e & Z_r \\ M_u & M_w & M_q & 0 & M_v & \begin{matrix} M_p - 2P_e I_{xz}/I_{yy} \\ -R_e (I_{xx} - I_{zz})/I_{yy} \end{matrix} & 0 & \begin{matrix} M_r + 2P_e I_{xz}/I_{yy} - \\ P_e (I_{xx} - I_{zz})/I_{yy} \end{matrix} \\ 0 & 0 & \cos\Phi_e & 0 & 0 & 0 & -\Omega_a \cos\Theta_e & -\sin\Phi_e \\ Y_u - R_e & Y_w + P_e & Y_q & -g\sin\Phi_e \sin\Theta_e & Y_v & Y_p + W_e & g\cos\Phi_e \cos\Theta_e & Y_r - U_e \\ L'_u & L'_w & L'_q + k_1 P_e - k_2 R_e & 0 & L'_v & {L_p}' + k_1 Q_e & 0 & L'_r - k_2 Q_e \\ 0 & 0 & \sin\Phi_e \tan\Theta_e & \Omega_a \sec\Theta_e & 0 & 1 & 0 & \cos\Phi_e \tan\Theta_e \\ N'_u & N'_w & N'_q - k_1 R_e - k_3 P_e & 0 & N'_v & {N_p}' & 0 & N'_r - k_1 Q_e \end{bmatrix}$$

$$B_i = \begin{bmatrix} X_{\theta_0} & X_{\theta_{1s}} & X_{\theta_{1c}} & X_{\theta_{0T}} \\ Z_{\theta_0} & Z_{\theta_{1s}} & Z_{\theta_{1c}} & Z_{\theta_{0T}} \\ M_{\theta_0} & M_{\theta_{1s}} & M_{\theta_{1c}} & M_{\theta_{0T}} \\ 0 & 0 & 0 & 0 \\ Y_{\theta_0} & Y_{\theta_{1s}} & Y_{\theta_{1c}} & Y_{\theta_{0T}} \\ {L_{\theta_0}}' & {L_{\theta_{1s}}}' & {L_{\theta_{1c}}}' & {L_{\theta_{0T}}}' \\ 0 & 0 & 0 & 0 \\ {N_{\theta_0}}' & {N_{\theta_{1s}}}' & {N_{\theta_{1c}}}' & {N_{\theta_{0T}}}' \end{bmatrix},$$

where u, v, w are components of the transitional velocity; p, q, r are components of the rotational velocity; φ, θ are Euler angles (roll, pitch angles) defining a position of the aircraft with respect to earth; θ_0 and θ_{0T} are the main and tail rotor collective pitch angles; θ_{1s} and θ_{1c} are longitudinal and lateral cyclic pitch; X, Y, Z and L, R, N are components of applied external forces and moments respectively, g is a gravitational acceleration, I_{xx}, I_{yy}, I_{zz} are moments of inertia about the corresponding axes; I_{xz} is a product of inertia about the x and z axes; $L_v, M_q,..$ are moment derivatives; $X_u, X_p,...$ are X - force derivatives; $Y_v, Y_r,...$ are Y - force derivatives; $Z_w, Z_q,...$ are Z - force derivatives; Θ_e, Φ_e are trim Euler angles; Ω_a is the aircraft angular velocity in trim flight; P_e, Q_e, R_e are trim angular velocities in fuselage axes system; U_e, V_e, W_e are trim velocities in fuselage axes system; k_1, k_2, k_3 are inertia coupling parameters; $X_{\theta_0}, X_{\theta_{1s}}, X_{\theta_{1c}}, Z_{\theta_0}, Z_{\theta_{1s}}, Z_{\theta_{1c}}, M_{\theta_0}, M_{\theta_{1s}}, M_{\theta_{1c}}$ are control derivatives regarding to main rotor longitudinal; $Y_{\theta_0}, Y_{\theta_{1s}}, Y_{\theta_{1c}}, L'_{\theta_0}, L'_{\theta_{1s}}, L'_{\theta_{1c}}, N'_{\theta_0}, N'_{\theta_{1s}}, N'_{\theta_{1c}}$ are control derivatives regarding to main rotor literal; $X_{\theta_{0T}}, Z_{\theta_{0T}}, M_{\theta_{0T}}, Y_{\theta_{0T}}, L'_{\theta_{0T}}, N'_{\theta_{0T}}$ are control derivatives regarding to tail rotor. The yaw angle is omitted because of the considered flight conditions.

Form the value functional (quality criterion) for each subsystem given above:

$$\|z(t)\|^2 = x^T(t) S_i\, x(t) + u^T(t)\, G_i\, u(t), \tag{2}$$

where $S_i \in R^{n\times n}$ is a nonnegative definite matrix, $G_i \in R^{k\times k}$ is a positive definite matrix.

Define L_2 gain of a subsystem as

$$\frac{\int_0^\infty \|z(t)\|^2 dt}{\int_0^\infty \|d(t)\|^2 dt} = \frac{\int_0^\infty [x^T(t) S_i\, x(t) + u^T(t) G_i\, u(t)] dt}{\int_0^\infty d^T(t) d(t)\, dt} \tag{3}$$

The purpose of the paper is to find the output feedback control $u(t)$ for the aggregated model (1) presented as the combination of subsystems for flight conditions with different values of the forward speed. The desired $u(t)$ should be designed so that the system is stable and L_2 gain is attenuated by some prescribed value γ^2.

3. Solution algorithm

The wished $u(t)$ is formed basing on the aggregation principle with use of output feedback control $u_i(t)$ for subsystems. It is proposed that $u_i(t)$ should be searched in the form

$$u_i(t) = -K_i^* C_i x(t) = F_i\, x(t),$$

where the gain K_i^* such that the i th subsystem is stable and L_2 gain is bounded by γ^2.

Then according to the result in [2] for the disturbance $d^*(t) = \frac{1}{\gamma^2} D_i^T P_i\, x(t)$ there exists K_i^* such that

$$K_i^* C_i = G_i^{-1}(B_i^T P_i + L_i),$$
$$L_i = G_i K_i^* C_i - B_i^T P_i,$$
$$A_i^T P_i + P_i A_i + \frac{1}{\gamma^2} P_i D_i D_i^T P_i - P_i B_i G_i^{-1} B_i^T P_i + S_i + L_i G_i^{-1} L_i = 0.$$

If it is required to solve the problem for a flight mode when the forward speed takes values differing from the mentioned above the interpolation concept based on using the basis of finite functions are applied. The utilized finite functions are [3,9]

$$S_p^*(\tau) = \begin{cases} 2^{p-1}(1+\tau)^p, \tau \in \left[-1, -\frac{1}{2}\right], \\ 1 - 2^{p-1}|\tau|^p, \tau \in \left[-\frac{1}{2}, \frac{1}{2}\right], \\ 2^{p-1}(1-\tau)^p, \tau \in \left[\frac{1}{2}, 1\right], \\ 0, \qquad \tau \notin [-1, 1] \end{cases}$$

where $p = 2,3,4$ is an interpolation parameter. The finite functions take zero values outside the interval $[-1,1]$.

Then the original aggregated system can be expressed in the form

$$\dot{x}(t) = (A + BF)x(t),$$

if the expansion of the system matrices in the basis of $S_p^*(\tau)$ are used: $A = \sum_i A_i S_p^* \left(\frac{V}{\Delta V_i} - (i-1) \right)$, $B = \sum_i B_i S_p^* \left(\frac{V}{\Delta V_i} - (i-1) \right)$, $F = \sum_i F_i S_p^* \left(\frac{V}{\Delta V_i} - (i-1) \right)$. Thus the output feedback $u(t) = -F\,x(t)$.

The detailed algorithm [2] is given below. It should be noted that this algorithms doesn't contain the step of setting initial gain K_i^0.

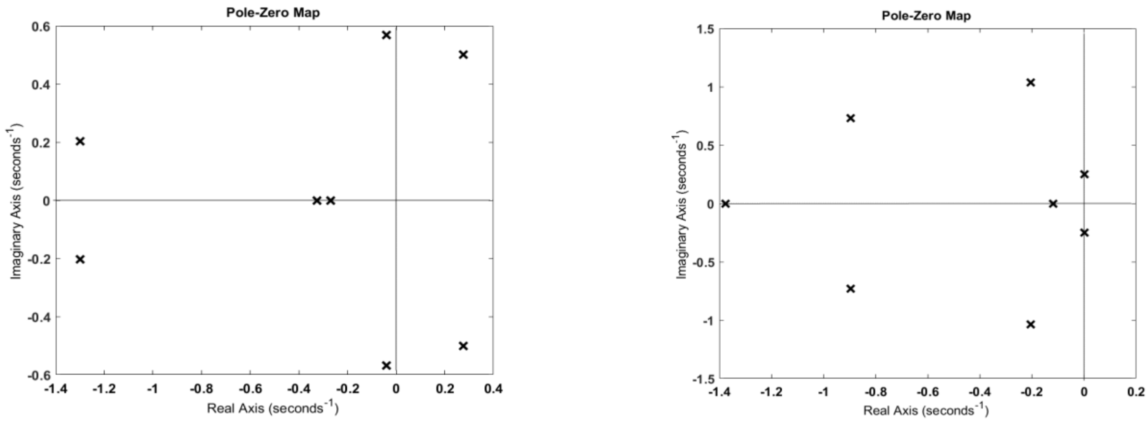
Algorithm

1. Set $k = 0,\ L_i^0 = 0,\ i = 1,..,m.$ Define small number ε.
2. Solve the Riccati equation
$$A_i^T P_i^k + P_i^k A_i + \frac{1}{\gamma^2} P_i^k D_i D_i^T P_i^k - P_i^{\,k} B_i G_i^{-1} B_i^T P_i^k + S_i + L_i^k G_i^{-1} L_i^{\,k} = 0$$
and find P_i^k.
3. Find $K_i^{k+1}, L_i^{k+1}, i = 1,..,m$:
$$K_i^{k+1} = Q_i^{-1}(B_i^T P_i^k + L_i^{\,k}) C_i^T (C_i C_i^T)^{-1},\ L_i^{k+1} = G_i K_i^{k+1} C_i - B_i^T P_i^k,$$
4. Stop and set $K_i^* = K_i^{k+1}$ if $\left\| K_i^{\,k+1} - K_i^{\,k} \right\| \le \varepsilon$. Otherwise set $k = k + 1$ and go to step 2.
5. Find $F_i = G_i^{-1} B_i^T P_i + L_i,\ i = 1,..,m$.
6. Find interpolated matrices A, B, F applying the expansion in the basis of the finite functions for a given mode.
7. Solve the equations of aircraft dynamics
$$\dot{x}(t) = (A + BF)x(t)$$
used any of numerical methods of appropriate order, for example the Runge-Kutta methods.

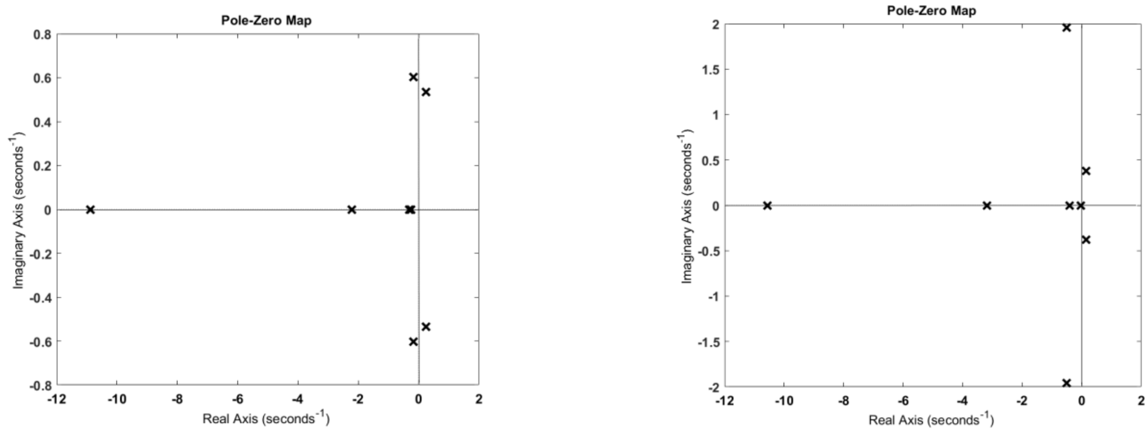
4. Analysis of helicopters' motion

The proposed technique is demonstrated on the example of modeling dynamics for three helicopter models: ZD559-Lynx, the twin engine, utility helicopter in the 4.5 -ton category; S123-Bo105, the twin engine helicopter in the 2.5 ton class; SA330-Puma, the twin engine, medium-support helicopter in the 6-ton category. Fig.1 visualizes poles location for three considered helicopters models before the H-infinity design is performed. It is clearly seen the aforementioned systems are originally unstable because some of poles are positioned on the right half plane and as consequence the synthesis of a regulator is required.

Figure 2 demonstrates the evolution of trajectories produced by the obtained control for three systems. The presented behavior of components of the vector state points out on damping transition processes and stabilizing systems. It should be noted that for the computation of the output feedback gain K_i^* the initialization is not needed. The parameter ε should be chosen in conformity with values of entries of system matrices. The γ^2-value is selected experimentally. The results are obtained provided that $0.2 \le \gamma^2 \le 0.4$ in dependence on a model of helicopter. It should be underlined that the very small γ^2-value is not acceptable because of impossibility of solving the Riccati equation. The closed-loop eigenvalues (λ_i) given in tables 1-3 for each of helicopter models verify the fact that the systems became stable. The solution algorithm is enough quickly implemented in MATLAB.



a)



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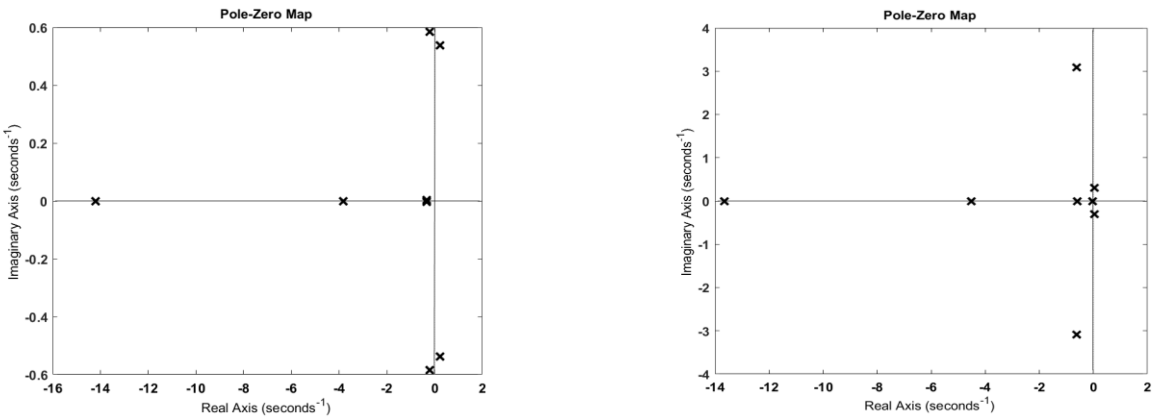

c)

Figure 1. Poles location for the helicopter systems: a) SA330-Puma b) ZD559-Lynx c) S123-Bo105 without the regulator if $V = 0$ (on the left) and $V = 40$ (on the right)

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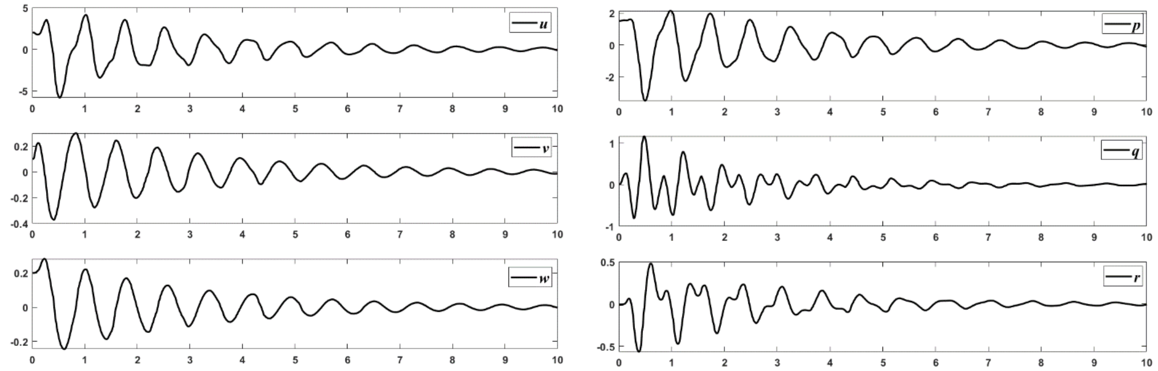
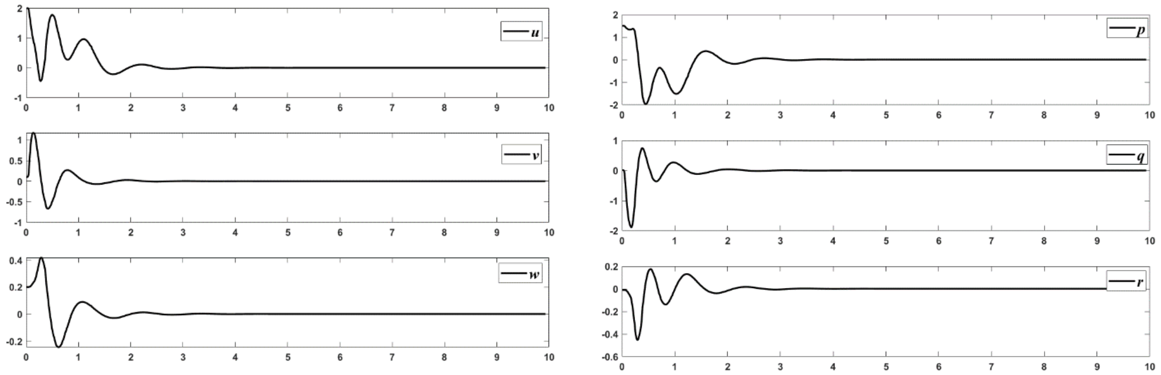


a)



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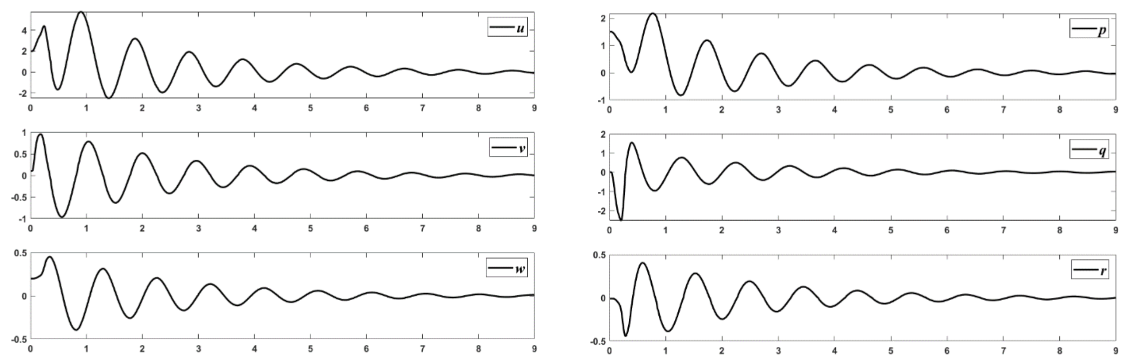

c)

Figure 2. Graphs of evolution of phase coordinates for a) SA330-Puma b) ZD559-Lynx c) S123-Bo105 if $V = 0$

Table 1. The closed-loop eigenvalues for SA330-Puma

V_i	0 (kts)	20 (kts)	40 (kts)	60 (kts)	80 (kts)	100 (kts)	120 (kts)	140 (kts)
	-1,20787	-1,15176	-1,13578	-1,06783	-1,0432	-1,03161	-1,02533	-1,02133
	-1,20787	-1,15176	-1,13745	-1,15431	-1,22637	-1,42084	-1,86443	-1,34891
	-2,12702	-1,47852	-1,13745	-1,15431	-1,22637	-1,42084	-1,86443	-4,32008
λ_i	-2,12702	-5,59646	-10,7964	-11,2438	-10,481	-9,82581	-9,07914	-7,731
	-11,1628	-11,9279	-10,7964	-13,9798	-17,7763	-20,6381	-22,764	-24,4443
	-11,9261	-11,9279	-14,1489	-17,4252	-20,6256	-24,0752	-27,9953	-33,0807
	-31,0569	-34,5725	-35,0073	-35,3097	-35,6274	-35,9804	-36,2652	-35,6286
	-120,933	-116,769	-125,582	-142,776	-159,393	-175,524	-191,877	-208,899

Table 2. The closed-loop eigenvalues for ZD559-Lynx

V_i	0 (kts)	20 (kts)	40 (kts)	60 (kts)	80 (kts)	100 (kts)	120 (kts)	140 (kts)
	-0.11889	-0.33728	-0.27424	-0.1652	-0,0721	-0,04907	-0,03786	-0,02224
	-0.65317	-2.34263	-1.92651	-1.78388	-1,92051	-2,03603	-2,27176	-2,77745
	-8.09829	-8.88193	-11.2115	-14.2157	-17,1709	-19,9274	-22,5653	-25,1583
	-11.0215	-8.88193	-11.2115	-14.2157	-17,1709	-19,9274	-22,5653	-25,1583
λ_i	-30.268	-24.544	-26.1545	-31.3221	-45,1835	-56,9563	-68,8041	-84,5709
	-30.268	-24.544	-26.1545	-31.3221	-45,1835	-56,9563	-68,8041	-84,5709
	-292.341	-460.351	-471.048	-563.535	-775,599	-888,523	-1007,12	-1170,21
	-1848.68	-832.02	-862.584	-931.541	-1185,1	-1272,59	-1348,46	-1464,44

Table 3. The closed-loop eigenvalues for S123-Bo105

V_i	0 (kts)	20 (kts)	40 (kts)	60 (kts)	80 (kts)	100 (kts)	120 (kts)	140 (kts)
	-0,65317	-2,34263	-1,92651	-1,78388	-1,92051	-2,03603	-2,27176	-2,77745
	-8,09829	-8,88193	-11,2115	-14,2157	-17,1709	-19,9274	-22,5653	-25,1583
	-11,0215	-8,88193	-11,2115	-14,2157	-17,1709	-19,9274	-22,5653	-25,1583
λ_i	-30,268	-24,544	-26,1545	-31,3221	-45,1835	-56,9563	-68,8041	-84,5709
	-30,268	-24,544	-26,1545	-31,3221	-45,1835	-56,9563	-68,8041	-84,5709
	-292,341	-460,351	-471,048	-563,535	-775,599	-888,523	-1007,12	-1170,21
	-1848,68	-832,02	-862,584	-931,541	-1185,1	-1272,59	-1348,46	-1464,44

In Figure 3 the graphs of evolution of phase coordinates for the case when the velocity changes linearly are given. To perform interpolation with use of the finite functions the value of the parameter p is 2. As is seen the results illustrate the effectiveness of the proposed solution algorithm. The system becomes stable. However it's worthwhile to say that the time which the system takes for damping processes increases (see Figure 2 b).

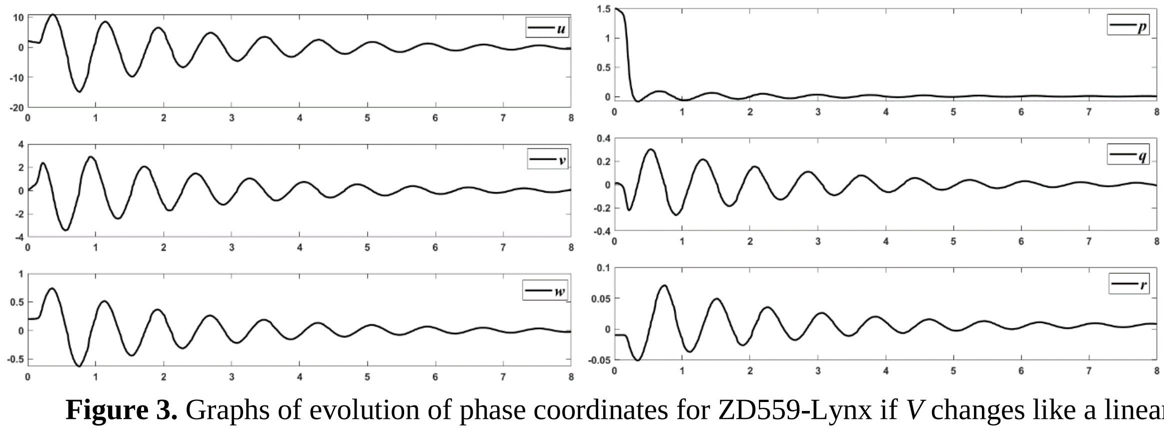

Figure 3. Graphs of evolution of phase coordinates for ZD559-Lynx if V changes like a linear function

5. Conclusion

In this paper, the H-infinity approach with combination of the interpolation principle is proposed to design the state feedback control of helicopter motion described by the aggregated model with uncertainties. The interpolation technique that consists in the use of the expansion of system matrices in the basis of finite functions permits to obtain the solution for modes of helicopters motion where the forward speed changes according to different types of functional correspondences. The described strategy is examined on the example of three helicopter models: ZD559-Lynx, S123-Bo105, SA330-

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Puma. The presented numerical results verify satisfaction of requirements for system transition processes.

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