ISSN 1068-7998, Russian Aeronautics, 2019, Vol. 62, No. 3, pp. 394–400. © Allerton Press, Inc., 2019.
Russian Text © The Author(s), 2019, published in Izvestiya Vysshikh Uchebnykh Zavedenii, Aviatsionnaya Tekhnika, 2019, No. 3, pp. 39–45.

FLIGHT DYNAMICS AND CONTROL OF FLIGHT VEHICLES

Developing the Mathematical Model of a Pilot in a Control Manual Preview Tracking Task

A. V. Efremov[a,*], M. S. Tyaglik[a], A. S. Tyaglik[a], and I. Kh. Irgaleev[a]

[a] *Moscow Aviation Institute (National Research University), Volokolamskoe sh. 4, Moscow, 125993 Russia*

**e-mail: pvl@mai.ru*

Received March 18, 2019; in final form, April 16, 2019

Abstract—This paper compares the pilot's control actions and the pilot–aircraft system characteristics obtained in the compensatory tracking tasks, tracking with pursuit and preview. The model of the pilot's control actions for all these cases is developed, and its adequacy to the experimental research results is investigated. By mathematical modeling the pilot–aircraft system, an optimal length of the corridor covering the planned trajectory is defined, which allows one to predict its further change. The result of modeling is compared with the results of experimental investigation on the simulator.

DOI: 10.3103/S1068799819030061

Keywords: pilot–aircraft system, manual control, predictive display, pursuit tracking task with preview.

In solving many applied problems associated with the choice of algorithms for the manual control system, display, and characteristics of controls, the pilot–aircraft system is interpreted as a compensatory system, in which the pilot perceives the error signal between the input signal and the aircraft response at the current time.

However, in some cases of piloting such an idea of the organization of a pilot–aircraft closed system is inaccurate. In particular, when aiming an aircraft at a target moving against the background of the Earth or when refueling in flight, the pilot can select the movement of the target or refueling cone, and in addition, he perceives an error signal. This case relates to the so-called pursuit tracking task, which is investigated in a number of papers [1], in which differences were found in the actions of the pilot compared with compensatory tracking.

When flying in the gorge, the pilot perceives not only the current position of the aircraft, but also the future planned trajectory. This task refers to the so-called preview manual task. In works [2–4] performed for a number of control objects $W_c = \frac{K}{p}$, $W_c = \frac{K}{p^2}$, the difference in the pilot's actions and the characteristics of the pilot–aircraft system in the tasks of compensatory tracking and preview tracking was shown. In the studies [5], when demonstration of the predicted path angle $\theta_{pr} = \theta + \dot{\theta}\frac{T_{pr}}{2}$ on the display screen (Fig. 1), the selection of the time T_{pr} was made on the basis of consideration of the aircraft–pilot compensatory system. This time is determined by the distance $L_{pr} = VT_{pr}$ between the pilot's eye and the *MN* plane (predictive window) located in the corridor, which covers the planned trajectory of motion, on which the velocity vector *b* is projected. The resulting recommendations showed that the introduction of such information significantly improves the control accuracy in the tracking task.

However, the quantitative indicators of the planned trajectory tracking accuracy and the variance of the control deflection obtained in the mathematical modeling differ from the experimental studies carried out on the ground-based simulator.

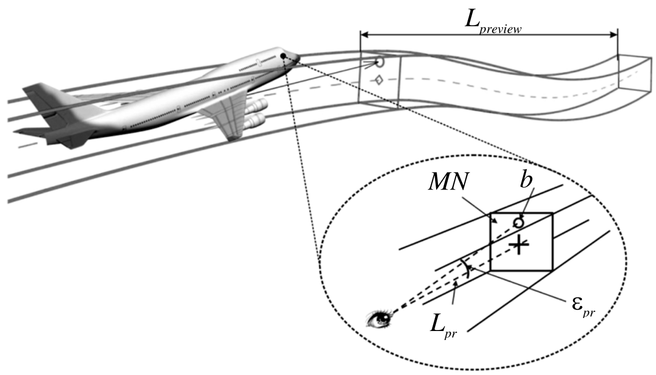


Fig. 1. A system of information display.

In this regard, this paper suggests a study of clarifying the model of the pilot behavior characteristics in the pursuit tracking task. This model was expanded to be used for investigation of preview manual control tracking tasks, in which the display shows a section of a planned trajectory of length $L_{preview} = VT_{preview}$ located beyond the predictive window. The model was used for mathematical modeling of the pilot–aircraft system to determine the optimal predictive time $T_{preview}$.

In [5, 6], the choice of the law for the prediction angle formation $\varepsilon_{pr} = \theta_{pr} + \frac{\Delta H}{L_{pr}}$ and the predictive time was defined by minimizing the dispersion of the error signal of the current aircraft position tracking relative to the planned trajectory $\sigma^2_{\Delta H}$.

In this case, the structural scheme shown in Fig. 2 was considered, where $\varepsilon_{pr} = \frac{\Delta H}{L_{pr}} + \theta + \dot{\theta}\frac{T_{pr}}{2}$,

$$W_c(p) = \frac{K_c\left(T_{pr}p^2 + 2p + \frac{2}{T_{pr}}\right)}{p^2\left(p^2 + 2\zeta\omega p + \omega^2\right)}.$$

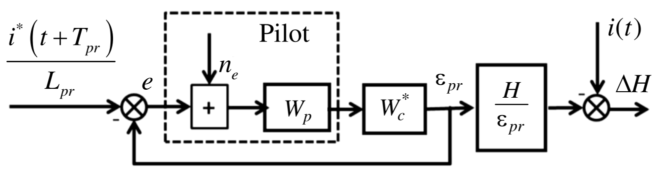
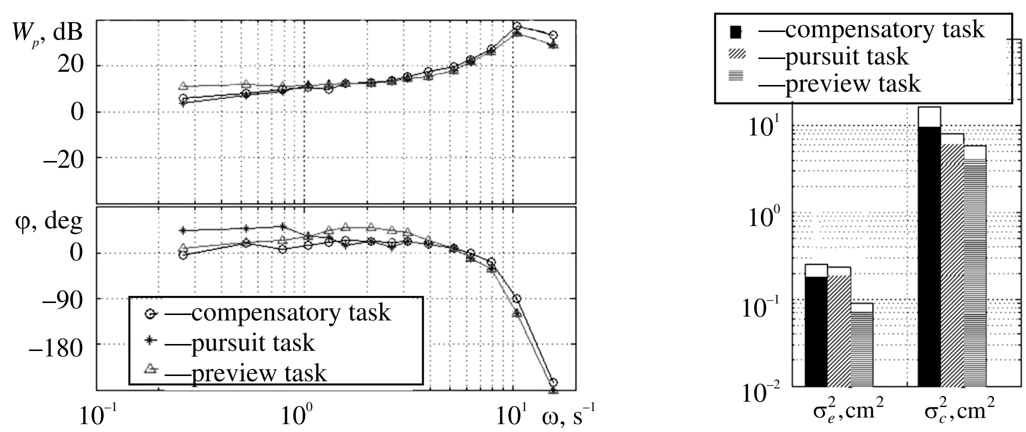


Fig. 2. Structural scheme of the pilot–aircraft system.

The study [5, 6] showed that in the absence of a prediction of a planned trajectory, that is $T^* = 0$, the dependence $\sigma^2_{\Delta H} = f(T_{pr})$ has the optimum at $T_{pr} = 0.7 - 1.0$ s. At the same time, the variance of error is three times less compared with the case of the absence of predictive information. Thus, the pilot's perception of the input signal of an error between the input signal $i(t+T_{pr})/L_{pr}$ and the predictive angle of view ε_{pr} can significantly improve the task performance. However, the interpretation of the pilot–aircraft system as a compensatory system is not entirely accurate, since when the corridor displays on the screen, the pilot perceives the input signal in addition to the error signal. His response to this signal transforms the compensatory system into a pursuit system. If the display shows information about the planned trajectory located beyond the predictive window, the pilot–aircraft system undergoes an additional transformation and becomes a system with preview.

In order to identify differences in the pilot's actions for these systems compared to a compensatory one, a series of experiments was conducted. Experimental studies were performed on the ground-based simulator of the pilot–aircraft system laboratory equipped with a wide-angle stereoscopic computer generated visual system.

For each of the three types of system (the compensatory system, the pursuit system (with mapping of the predictive window and predictive velocity vector), and the pursuit system with preview (with mapping of the predictive window, three-dimensional corridor, and predictive velocity vector)), ten experiments were performed, in which the task was assigned to the pilot to minimize the error signal generated by the information display system proposed in [5, 6]. The results of the experiments were processed in order to obtain the frequency characteristics of the pilot, as well as variances of the error signal and the control deflections in accordance with the software given in [7, 8]. It is seen from the studies results shown in Fig. 3 that in the case of transition from a compensatory system to a pursuit system, the accuracy of the piloting task does not practically change, but this decreases the average deflection of the controls (dispersion decreases by more than 2 times). This is due to the increase in the phase response of the pilot in the low-frequency region. When we transit to the pursuit system with preview, a significant improvement in the task performance (3 times) occurs. At the same time, some reduction in the controls deflection took place. The amplitude frequency response of the pilot increases in the low and crossover frequency band, and the frequency response increases in the crossover frequency band.



(a) (b)

Fig. 3. Pilot frequency response characteristics (a), variance of the error signals and the control deflection (b).

In a separate series of experiments, the length of the tunnel covering the planned trajectory was changed on the display screen along with the display of the predictive window with projection of the predicted velocity vector.

The purpose of these experiments was to determine the length of the corridor $L_{preview} = VT_{preview}$ (see Fig. 1) located beyond the predictive window *MN*. Obviously, a short length will not give the pilot enough information, and a significant length of the corridor will either be useless or cause difficulty in perception of a complex program trajectory. As a result of these experiments, the optimal length of the trajectory display $T_{preview}$ on the screen was determined (Fig. 4).

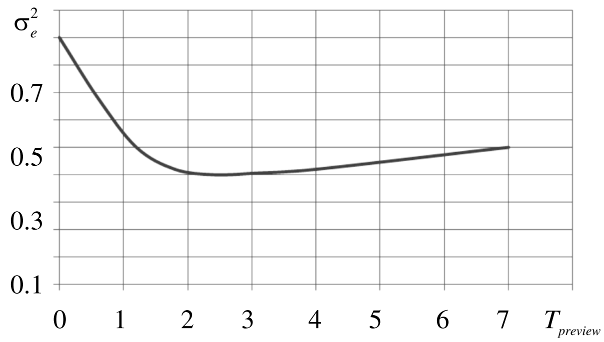


Fig. 5. Effect of the preview time on piloting accuracy.

This shows that the visualization of a considerable length of the planned trajectory section does not increase the accuracy of piloting, but even worsens it somewhat.

PILOT BEHAVIOR MODEL

Currently, several approaches are widely used to model pilot control behavior, namely, structural, optimal, neural network, and using the fuzzy logic [9].

In order to obtain a sufficient general methodology for optimizing the predictive time, a mathematical model was developed for the pilot's control response characteristics, describing the behavior of the pilot in the pursuit and pursuit with preview tracking tasks. To account for the perception of information in the task of pursuit and preview, an additional signal was formed, which is summed with the error signal $e(t)$. In the control task with the pursuit, a signal, which is generated as the signal $i(t)$ passed through a filter like $F = \frac{K_0 p}{(T_{tr} p + 1)}$, is introduced as an additional signal $\alpha_1(t)$ summed with the error signal.

The time constant T_{tr} was chosen from the condition of proximity of the frequency response of the function describing the pilot that was obtained in the experiment and in mathematical modeling (in the present paper $T_{tr} = 0.1$), and the coefficient K_0 was chosen on the basis of the minimum error variance.

In the case of preview tracking task, an additional signal $\alpha_2(t) = K_1 \frac{[i(t+\Delta t) - i(t)]}{\Delta t V} + K_2 \frac{[i(t+2\Delta t) - i(t)]}{\Delta t V} + \ldots$ was introduced (Fig. 5). It is summed up with the error signal $e(t)$ too.

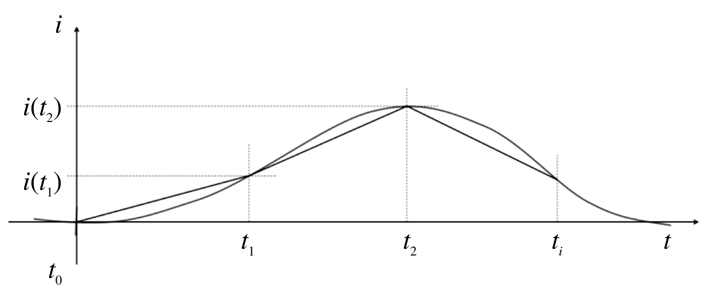


Fig. 5. Additional signal $\alpha_2(t)$.

This signal is essentially a weighted sum of slopes of the trajectories $i(t)$ in the sections of the same length ΔtV (see Fig. 5) and determines the process of perception of the future program trajectory by the pilot. The weight coefficients K_i (i = 1, 2, ... n) determine the degree of importance of the information perceived by the pilot, located at different distances from the predictive window.

The developed mathematical model of the pilot's behavior for the pursuit task and the pursuit task with preview is presented in Fig. 6. Here, the pilot's perception of the input signal in the pursuit task and the future planned trajectory in the pursuit task with preview is modeled using an additional signal $\alpha_{av}(t) = \alpha_1(t) + \alpha_2(t)$. This model is the Hess modified model [10].

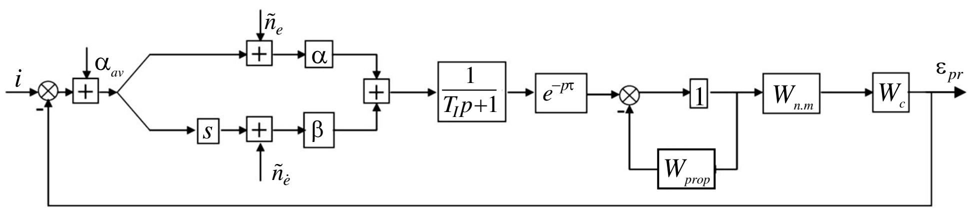


Fig. 6. Structural scheme of pilot model in the task of prediction and preview.

Here the models of spectral noise densities $\tilde{n}_e$ and $\tilde{n}_{\dot{e}}$ take into account the influence of dead zones; $e^{-p\tau}$ is the element that takes into account the delay in the information perception; W_{prop} is the block characterizing the pilot's adaptation to proprioceptive information; $W_{n.m}$ is the dynamics of the neuromuscular system.

The choice of parameters of the pilot's control actions model ($K_L = \alpha$, $T_L = \frac{\alpha}{\beta}$, K_n, T_n) was implemented by minimizing the error variance $\sigma_e^2 = \sigma_{e_n}^2 + \sigma_{e_i}^2$, where $\sigma_{e_i}^2$ is the error signal share determined by the input signal, and $\sigma_{e_n}^2$ is the error variance share determined by the remnant calculated in accordance with the expression $\sigma_{e_n}^2 = \frac{\sigma_{e_i}^2 + T_L^2 \sigma_{\dot{e}_i}^2}{\frac{1}{K_{n_e}} - \int_0^\infty |\Phi(j\omega)|^2 \, d\omega} \int_0^\infty \frac{|\Phi(j\omega)|^2}{1 + T_L^2\omega^2} d\omega$ [8], where $\Phi(j\omega)$ is the describing function of the pilot–aircraft closed-loop system in regard to the tracking error.

The procedure for selecting the parameters that determine the response of the pilot to the visual and proprioceptive information in the absence of an additional signal $\alpha_{av}(t)$ and in the case of a random input signal $i(t)$ is given in [8].

For the case in question, the choice of parameters of the pilot model including the coefficients K_i, which are the part of expression for the signal $\alpha_{av}(t)$, should be refined.

Due to the fact that the signal $\alpha_{av}(t)$ is a non-random continuous input signal characterized by spectral density, the characteristics of the pilot's control actions were determined by numerical integration, rather than using a system of algebraic equations, whose elements require the calculations of improper integrals [7, 11].

At the first step of the algorithm, the input signal $i(t)$ is calculated, simultaneously with it, a signal $i^*(t, t + \Delta t, t + 2\Delta t, ...)$ is calculated that describes the change of the planned trajectory to the moment

$t + n\Delta t$ and allows calculating the additional signal $\alpha_2(i^*)$ value. The input signal $i(t)$ and the output coordinate $y(t)$, as well as an additional signal $\alpha_2(i^*)$, form the signal $e(t) + \alpha_2(t)$ perceived by the pilot.

Since the input signal $i(t)$ is a discrete polyharmonic signal (a fixed set of frequencies and amplitudes) calculated over a finite time interval $T_{pr} < t < t_k$ determined by the lowest frequency period. In this regard, the numerical integration of the linear part of the pilot–aircraft system carried out over the time interval $T_{pr} < t < t_k$ will allow calculating the signals $e(t)$, $c(t)$, $y(t)$ and the error variance $\sigma^2_{e_i}$ of the linear part of the pilot–aircraft system for the initial combination of the pilot model parameters K_L, T_L, K_n, T_n.

After calculating, the variance of the total error σ^2_e is minimized. This procedure is based on a multi-parameter global optimization of the total error variance value, where the parameters of the pilot model K_L, T_L, K_n, T_n are used as optimization parameters.

In the subsequent steps of the procedure, the parameters K_L, T_L, K_n, T_n are updated, and the variances of the total error σ^2_e are calculated, after which the correspondence of σ^2_e to the minimum value is checked. If this condition is not met, the procedure is performed again with the new parameters of the pilot model.

The weight coefficients of the additional signal K_i were calculated sequentially. First the coefficient K_1 was entered and its value was selected based on the minimum error variance. Then the second term was added to the expression $i(t)$ and its value was determined, etc.

As a result, the minimum value of the variance of the total error and parameters of the pilot model and coefficients K_i corresponding to it are determined for calculating the frequency responses describing the pilot functions.

The pilot frequency responses calculated by using them demonstrate a fairly good agreement with the frequency responses (Fig. 7) obtained in experimental investigations carried out on the ground-based simulator.

Figure 8 demonstrates the dependence of piloting accuracy on time $T_{preview}$ for the calculated values K_i.

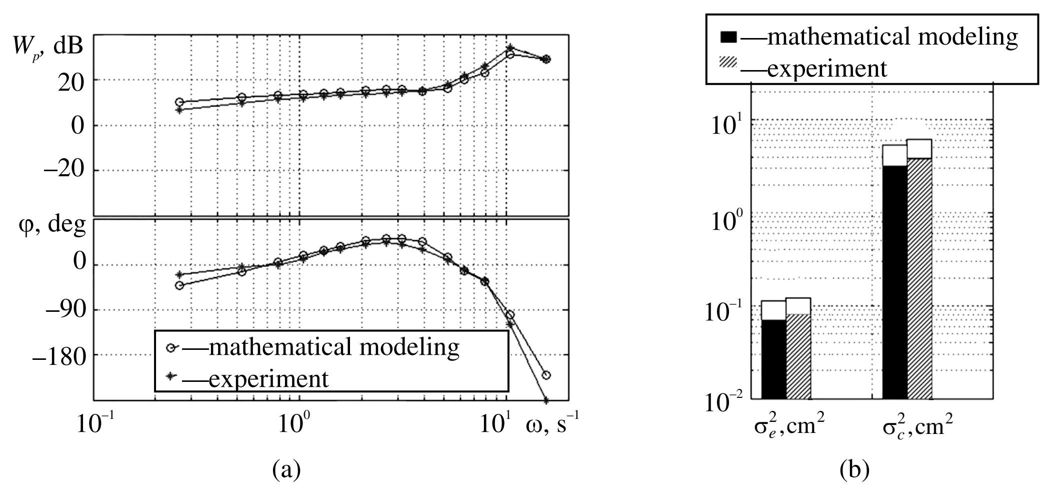


Fig. 7. Comparison of the frequency (a) and integral (b) characteristics.

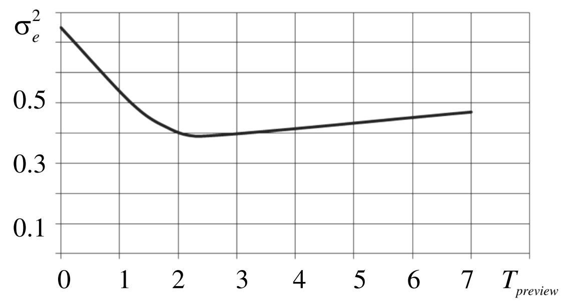


Fig. 8. Determining the preview time using the mathematical modeling.

It is seen that, just like in the experiment (see Fig. 4), an increase in time $T_{preview}$ leads to a significant decrease in the variance of the tracking error of the current trajectory position down to $T^* \cong 2.5$ s. Further growth of $T_{preview}$ does not lead to a decrease in the error, and even causes, as in the experiment, a slight increase in the variance of the tracking error. This result indicates that the considerable length of the planned trajectory displayed on the screen reduces the importance of this information for the pilot.

ACKNOWLEDGEMENTS

The work was carried out in the framework of the basic part of the state order no. 9.7465.2017/8.

REFERENCES

1. Wasicko, R.J., McRuer, D.T., and Magdaleno, R.E., *Human Pilot Dynamic Response in Single-Loop Systems with Compensatory and Pursuit Displays*, Wright-Patterson AFB, 1966, Tech. Rep. AFFDL-TR-66-137.
2. Reid, L.D. and Drewell, N.H., A Pilot Model for Tracking with Preview, *Proc. 8th Conf. on Manual Control*, Ann Arbor, USA, 1972, pp. 191–204.
3. Van der El, K., Padmos, S., Pool, D.M., van Paassen, M.M., and Mulder, M., Effects of Preview Time in Manual Tracking Tasks, *IEEE Transactions on Human-Machine Systems*, 2018, vol. 48, issue 5, pp. 486–495.
4. Van der El, K., Pool, D.M., van Paassen, M.M., and Mulder, M., Effects of Preview on Human Control Behavior in Tracking Tasks with Various Controlled Elements, *IEEE Transactions on Cybernetics*, 2017, vol. 48, issue 4, pp. 1242–1252.
5. Efremov, A.V. and Tyaglik, M.S., The Development of Perspective Displays for Highly Precise Tracking Tasks, *Aerospace Guidance, Navigation and Control, Selected Papers of the 1st CEAS Specialist Conference on Guidance, Navigation and Control*, Germany, Springer, 2011, pp. 163–174.
6. Garkushenko, V.I. and Lazareva, P.A., On the Problem of Improving the Flight Safety in Manual Control of the Aircraft, *Izv. Vuz. Av. Tekhnika*, 2017, vol. 60, no. 4, pp. 46–50 [Russian Aeronautics (Engl. Transl.), vol. 60, no. 4, pp. 528–533].
7. Efremov, A.V., et al., *Investigation of Pilot Induced Oscillation Tendency and Prediction Criteria Development*, USA, Wright Patterson AFB, Final report WL-TR-96-3109, 1996, URL: https://archive.org/details/DTIC_ADA322290/page/n5.
8. Efremov, A.V., Ogloblin, A.V., Predtechinskii, A.N., and Rodchenko, V.V., *Letchik kak dinamicheskaya sistema* (A Pilot as a Dynamic System), Moscow: Mashinostroenie, 1992.
9. Vereshchikov, D.V., Voloshin, V.A., Ivashkov, S.S., and Vasil'ev, D.V., Applying the Fuzzy Logic for Developing a Simulation Model of Pilot's Control Actions, *Trudy MAI*, 2018, no. 99, URL: http://trudymai.ru/eng/published.php?ID=91926.
10. Hess, R., Unified Theory for Aircraft Handling Qualities and Adverse Aircraft Pilot Coupling, *J. of Guidance, Control, and Dynamics*, 1977, vol. 20, no. 6, pp. 1141–1148.
11. Efremov, A.V., Aleksandrov, V.V., Efremov, E.V., and Vukolov, M.V., The Influence of Different Types of Inceptors and Their Characteristics on the Pilot-Aircraft System, *2nd IFAC Conference on Cyber-Physical and Human Systems CPHS 2018*, Miami, USA, *IFAC-PapersOnLine*, 2018, vol. 51, issue 34, pp. 372–377.