MOSCOW AVIATION INSTITUTE

Course: Flight Control

(part 1)

Dean of aeronautical school

Prof. A.V. Efremov, Ph. D

Any created technical system (its features and characteristics) is defined by customer requirements + existed potentialities

Technical and scientific potentialities:

– achievements in aerodynamics, materials, propulsion;

- subsystems design and technology (computers, actuators, avionics, etc).

Customer requirements are defined by the challenges

Challenges are different in different historical periods

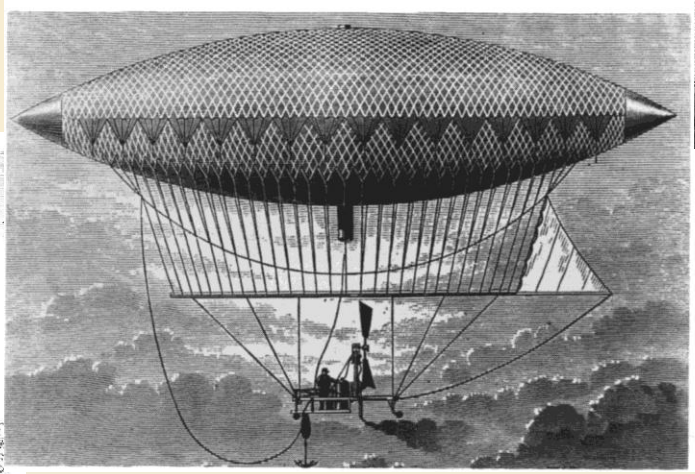
PREHISTORY ERA

Challenge – just to fly

To fly … ancient dream of man

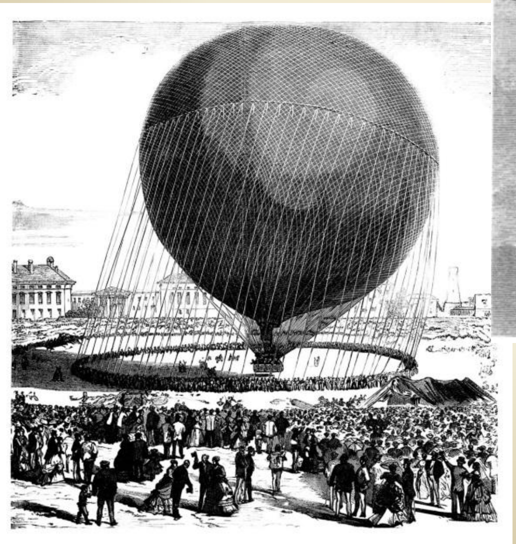
Giffard

1852



Mongolfier

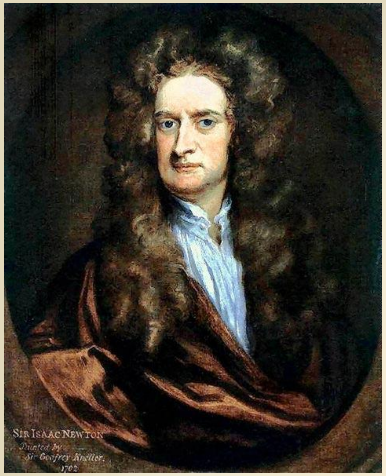
1783



Lilienthal

1891

FUNDAMENTAL BASIS DEVELOPED IN THE PAST

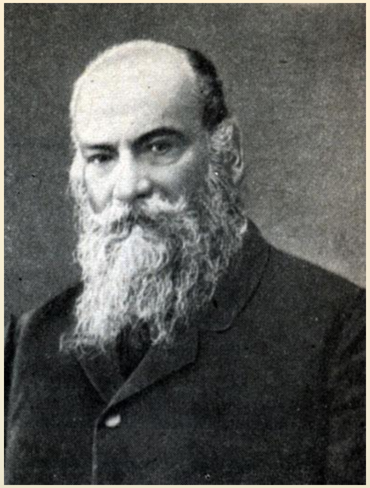


I. Newton

L. Euler

D. Bernulli

A. Penaud

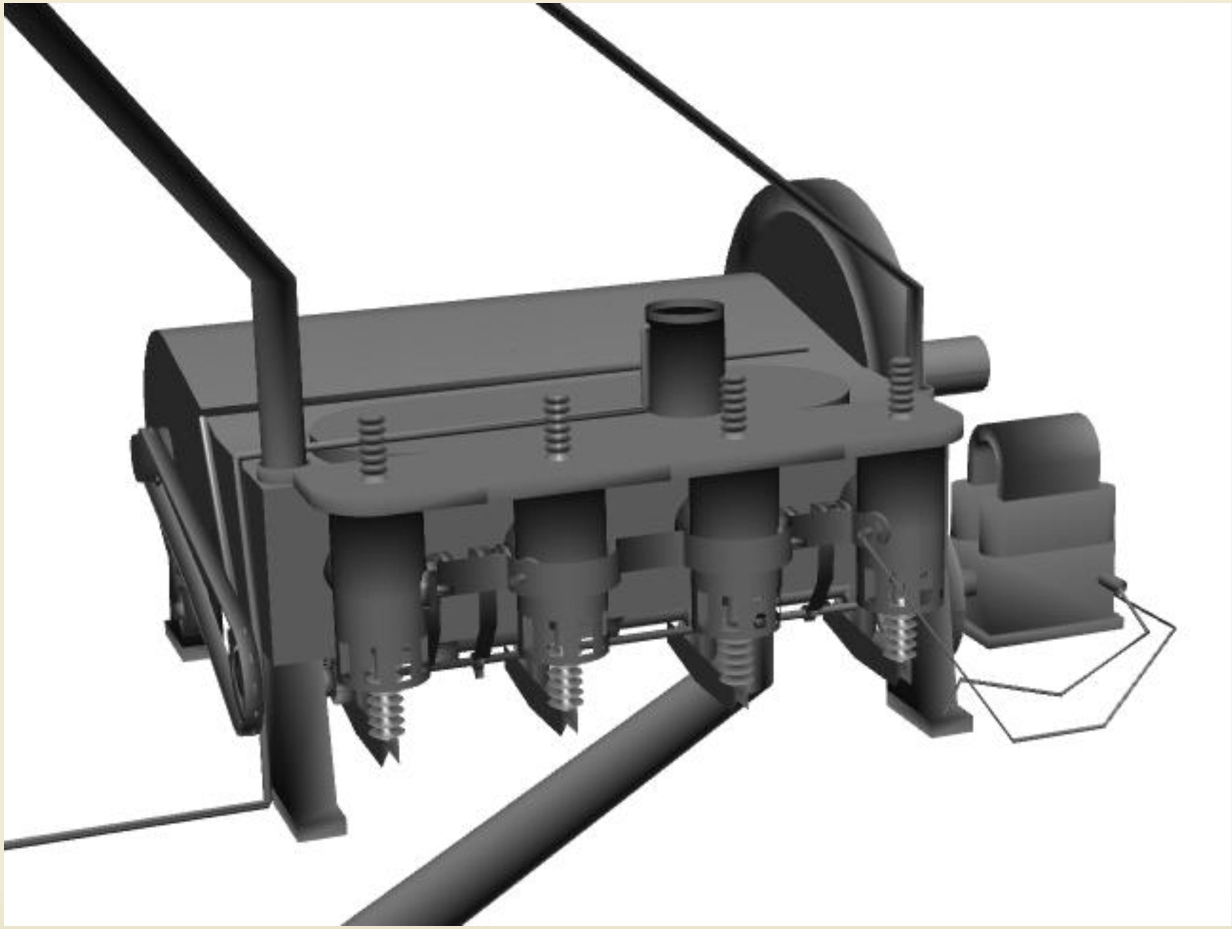


N. Zhukovsky

L. Prandtl

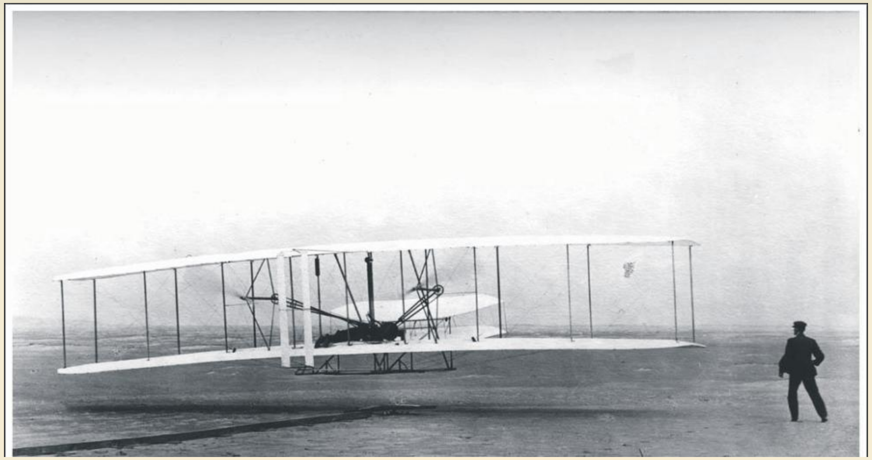
POWERED FLIGHT

Piston engine era



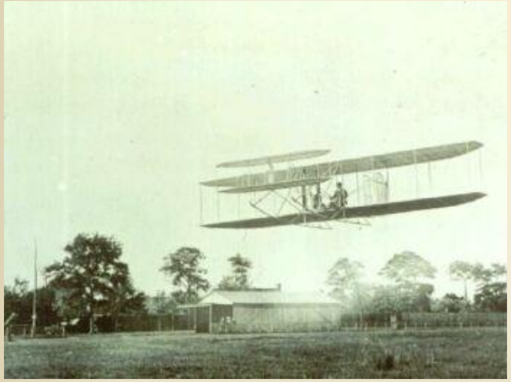
The first piston engine for Wright brother’s flyer

STAGE OF “TRIALS AND ERRORS TECHNIQUE”



**Wright brothers
1903**

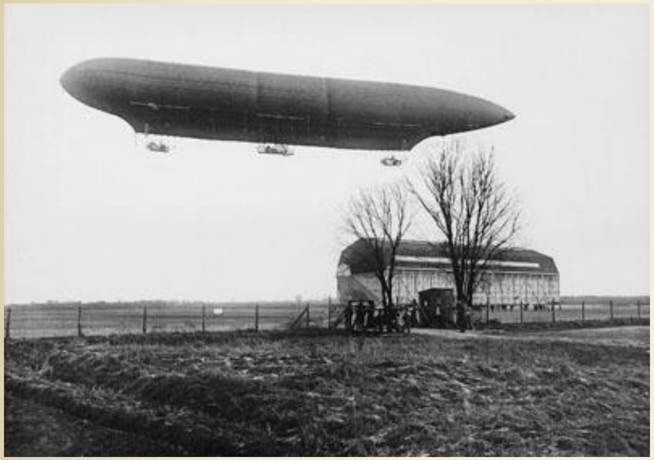
**Trials and error technique is based on experience
and similarity of configuration**



MILITARY AIRCRAFT IN THE FIRST WORLD WAR

COMMERCIAL VEHICLES

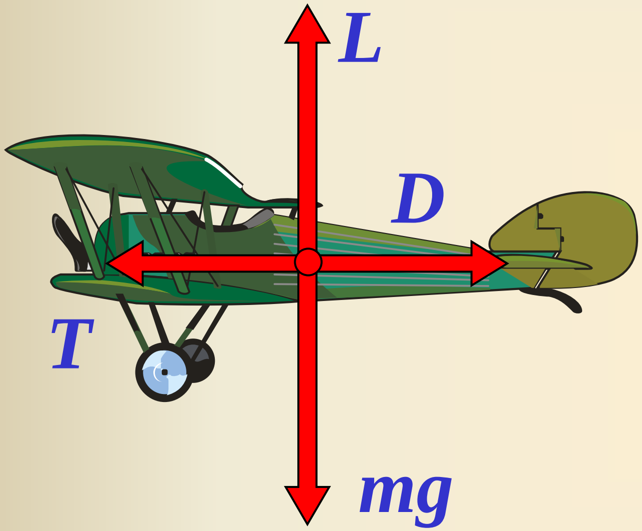
– **Airships (Zeppelin, 1900)**



– **Commercial Aircraft**

APPLIED SCIENCES AS THE BASIS FOR TECHNIQUE USED IN AIRCRAFT DESIGN

Flight mechanics



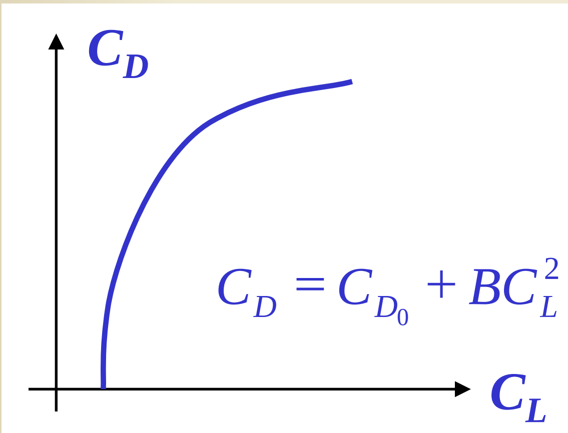
$$m\frac{d\overline{V}}{dt} = \overline{F} \qquad \frac{d\overline{Q}}{dt} = \overline{M}$$

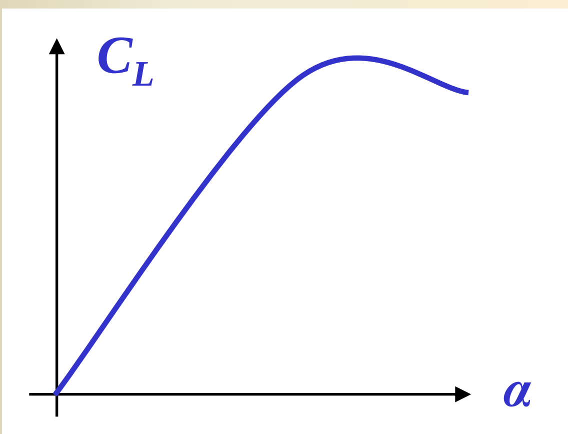
$$\left.\begin{matrix} L = W \\ T = D \end{matrix}\right\} \Rightarrow T = \frac{W}{L/D}$$

where

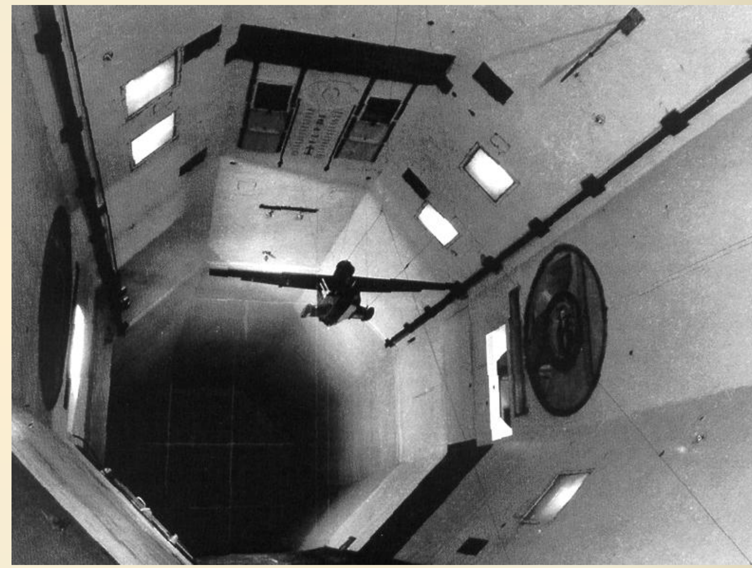
$$\left.\begin{matrix} L = C_L \dfrac{\rho V^2}{2} S \\ D = C_D \dfrac{\rho V^2}{2} S \end{matrix}\right\} \Rightarrow L/D = \frac{C_L}{C_D}$$

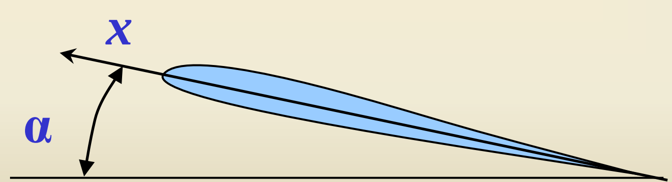
AERODYNAMICS





Wind tunnel
(TSAGI,Russia, 1925)





PROPULSION

Propeller Theory → size, form of propeller, power P_{pr}

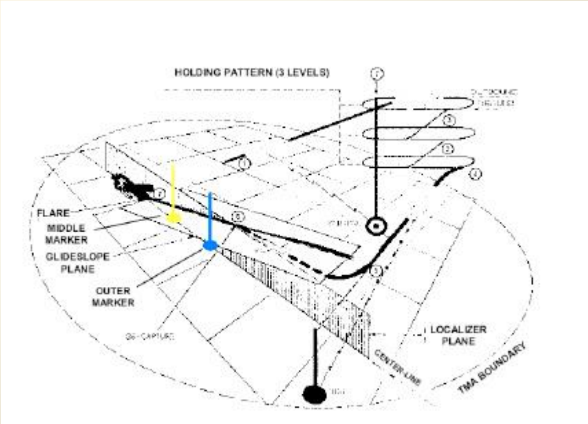
$$P = \eta \times P_{pr} = T \times V$$

NAVIGATION AND INSTRUMENTS

Sensors:
– first gyros and autopilot
(Sperry, 1911)

Radar:
(Radio Detection and Ranging)
(Watson Watt, 1935)

ILS:
(Instrument Landing System)
(GB, Germany, 1930th)



Technique for **flight performances** estimation and selection of main aircraft parameters

Customer requirements

↓

T →

L →

D →

Technique for flight performances estimation

$$V_{\min} = \sqrt{\frac{2W}{C_L \rho S}}; \quad \dot{h} = \frac{T - D}{W} V$$

$$R = f\left(\frac{W}{S}; \frac{C_L}{C_D}; \ldots\right); \quad \omega = \frac{g\sqrt{n^2 - 1}}{V}$$

•••••••••••••••

→ **aircraft parameters and characteristics**

Improvement of flight performances

$$\frac{W}{S} \Uparrow; \; C_L \Uparrow; \; \frac{C_L}{C_D} \Uparrow; \; \frac{T}{W} \text{ or } \frac{P}{W} \Uparrow; \; C_D \Downarrow$$

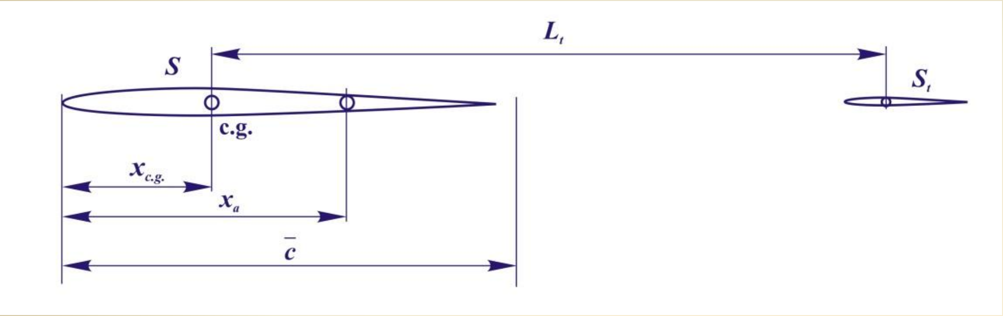
FLYING QUALITIES

(characteristics of stability and controllability)

"Men already know how to construct wings or airplanes…, how to build engines and screws of sufficient lightness and power…Inability to balance and steer still confronts students of the flying problem… <u>When this one feature has been worked out, the age of flying machines will have arrived</u> ."

W. Wright (1901)

Stability – aircraft feature to return back after disturbance



installation of stab causes the increase of stability

for $M < 1$ ➡ $x_a = const$

$\bar{x}_{c.g.} - \bar{x}_a < 0 \quad \left(C_{m_\alpha} < 0\right)$ ⬅ feature of stability

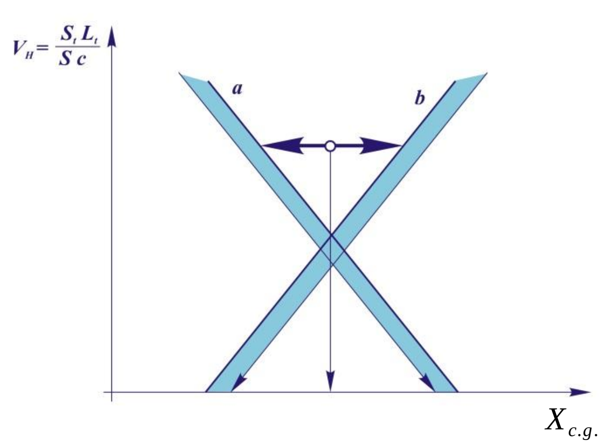
Controllability (handling qualities) – aircraft feature to respond (by corresponding way) on pilot's input

Handling qualities requirements of that period

1) to balance aircraft $\Sigma M = 0$ ($\Sigma\, Cm = 0$)

2) to provide reasonable forces applied by pilot to wheel

The technique developed in 30th – 40th for provision of flying qualities requirements



⟷ – interval of c.g. variation

REQUIREMENTS:

a) aircraft balance (trim) in horizontal flight

$$\Sigma C_m = 0 \qquad (\Sigma m_z = 0)$$

b) stability margin

$$\frac{\partial C_m}{\partial C_L} = \bar{x}_{c.g.} - \bar{x}_a \leq \Delta$$

$$\left(\frac{\partial m_z}{\partial C_y} = \bar{x}_{ц.m.} - \bar{x}_F \leq \Delta\right)$$

calculation of aerodynamic coefficients or wind–tunnel test → reliable definition of stability

independence of x_a on Mach number → + ← reliable definition of stability

+ → guarantee of necessary controllability

END OF PISTON ENGINE ERA

To the end of II World War

$$V_{max} \sim 700 \div 750 \ km/h$$

Piston engine does not allow to increase these velocities

$$T = \frac{\eta \times P_{pr}}{V} \Leftrightarrow D = f(V^2)$$

At the end of II World War
the jet engine aircraft was created

JET AVIATION ERA

MILITARY AVIATION

Me-262, 1943

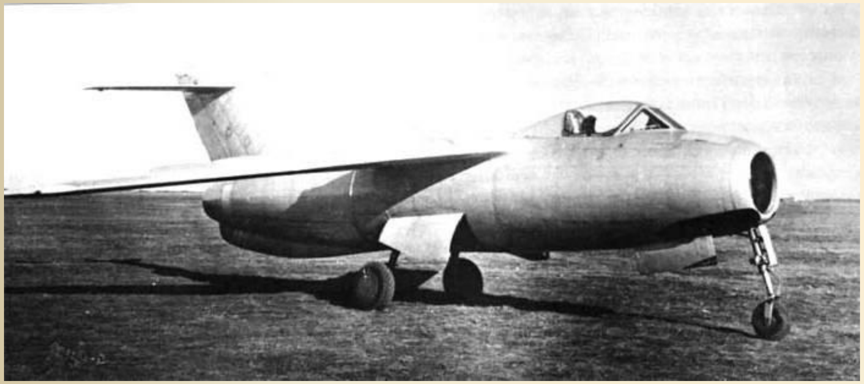
PASSANGER AVIATION

“Comet”, 1952

SUPERSONIC FLIGHT

First supersonic flights

F-86 “Saber” (USA), 1947
(supersonic velocity was reached in decent)

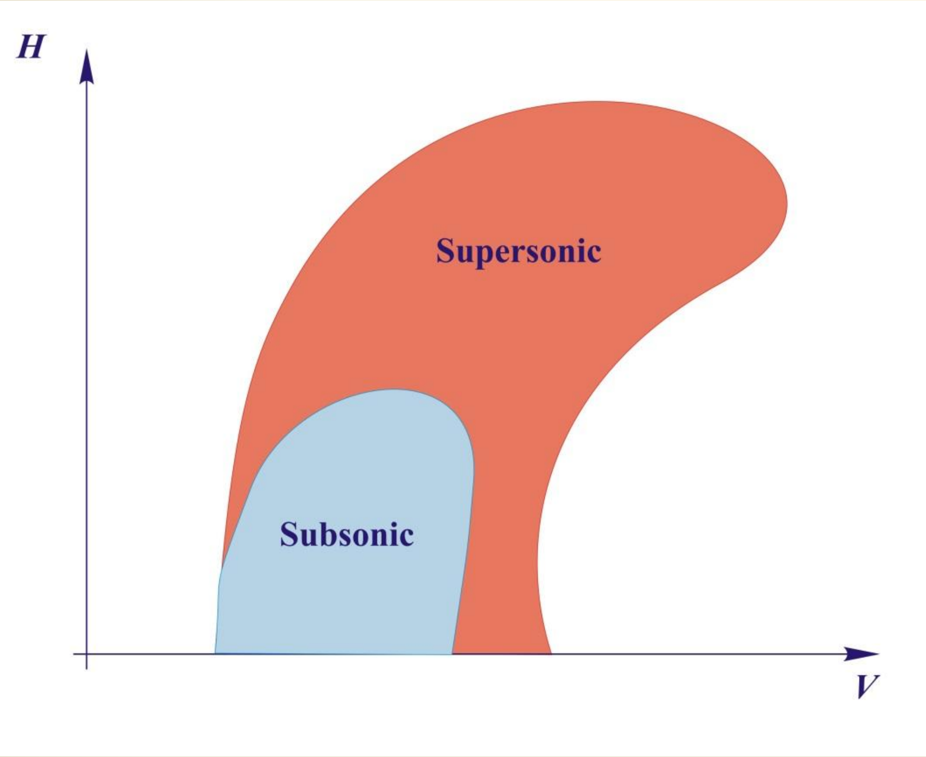


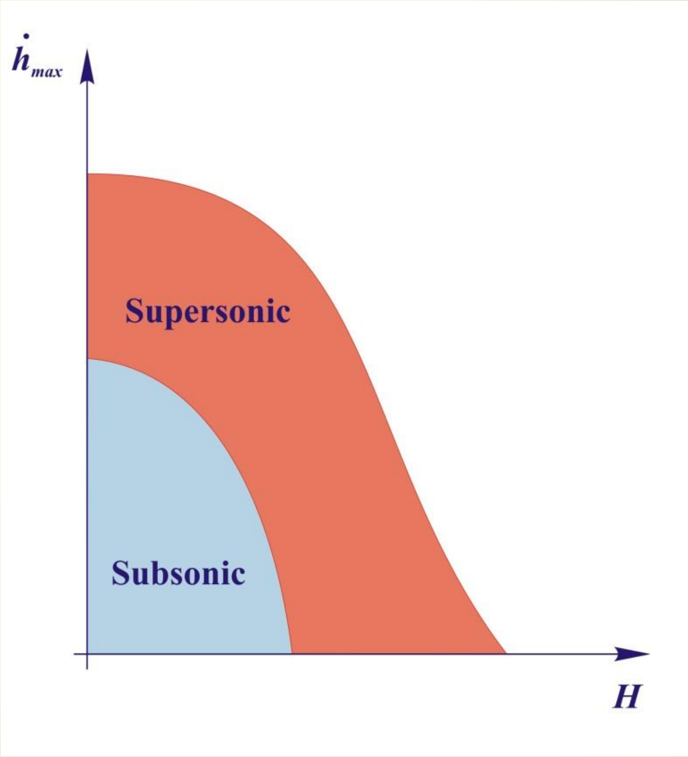
La–176 (USSR), 1948
(supersonic velocity was reached in decent)

MiG–17 (USSR), 1950
(supersonic velocity was reached in horizontal flight)

Potentialities of supersonic aircraft

Considerable improvement of flight performance

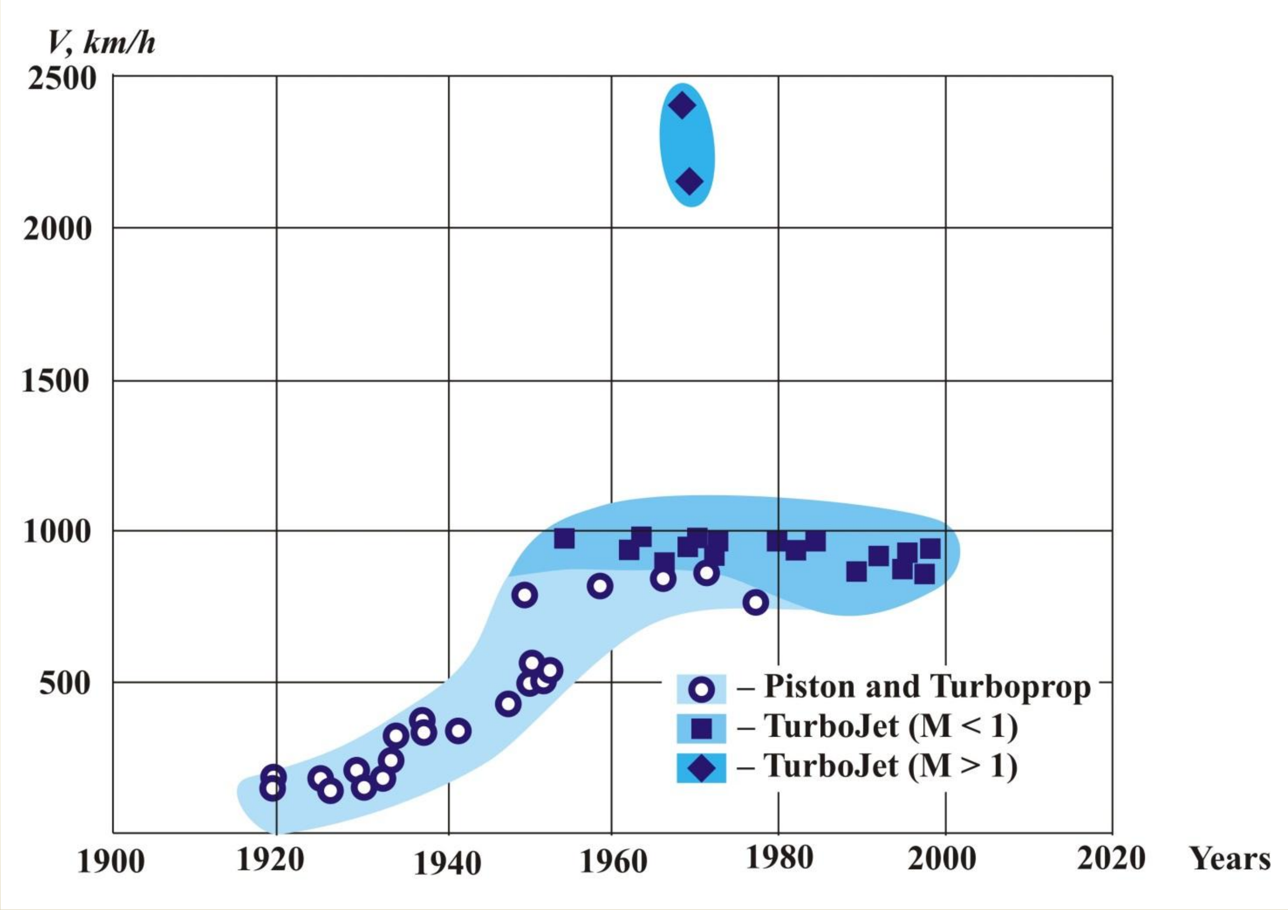




Jet passenger and transport aviation

General features:

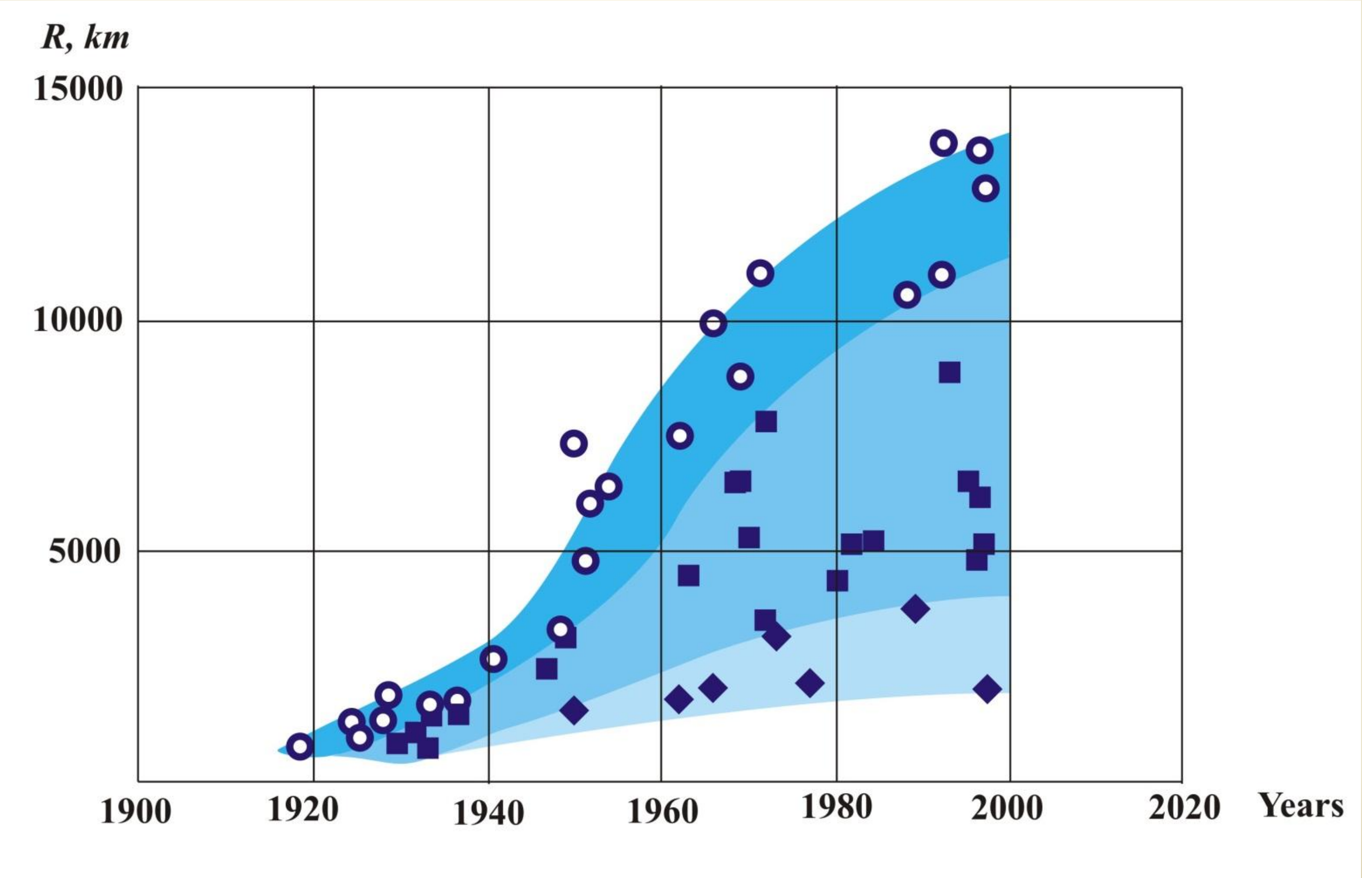
Increase of velocity



Jet passenger and transport aviation

General features:

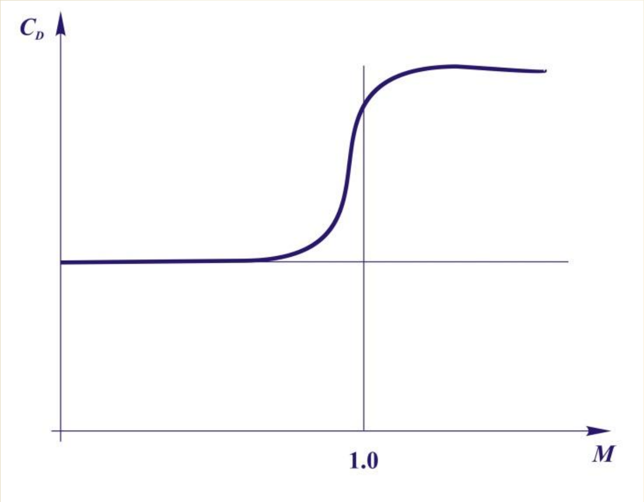
Increase of range



Problems of supersonic flight

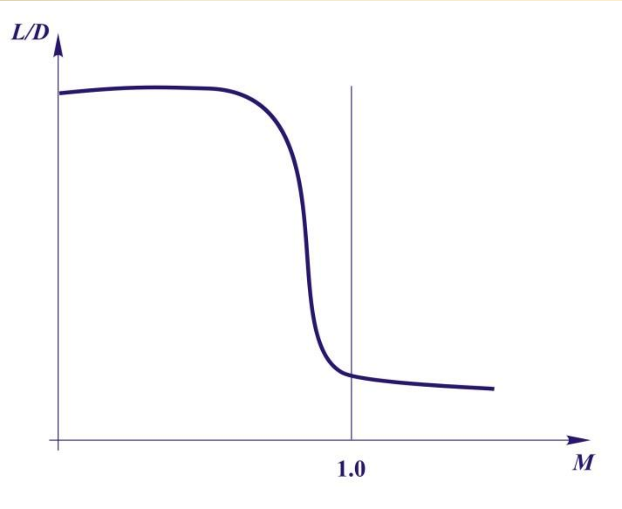
Aerodynamic forces

● **increase of C_D**



– increase of required thrust

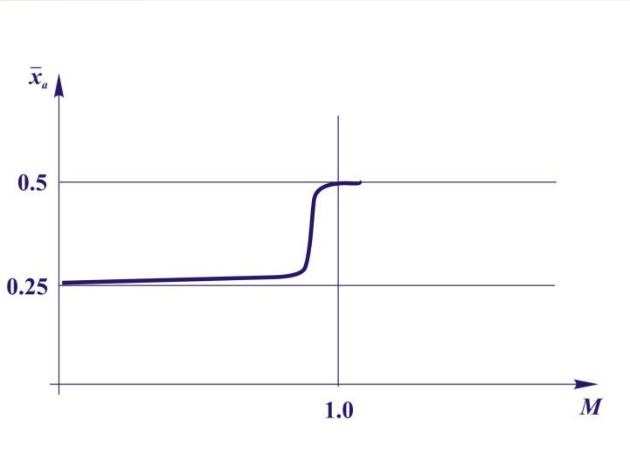
● **decrease of *L/D* ratio**

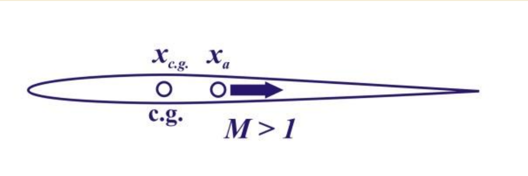


– decrease of range

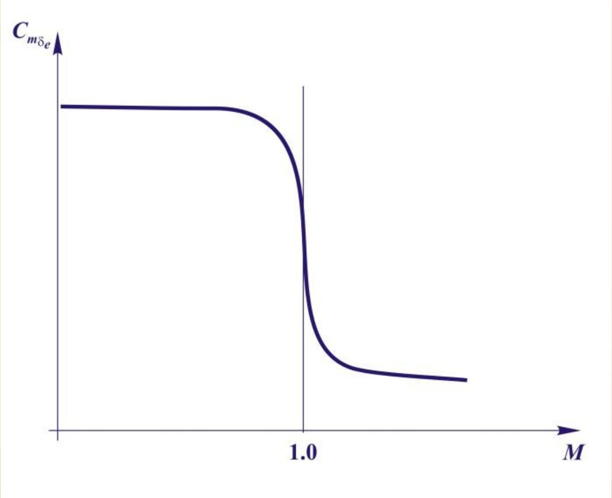
Aerodynamic moments

– change of aerodynamic center location





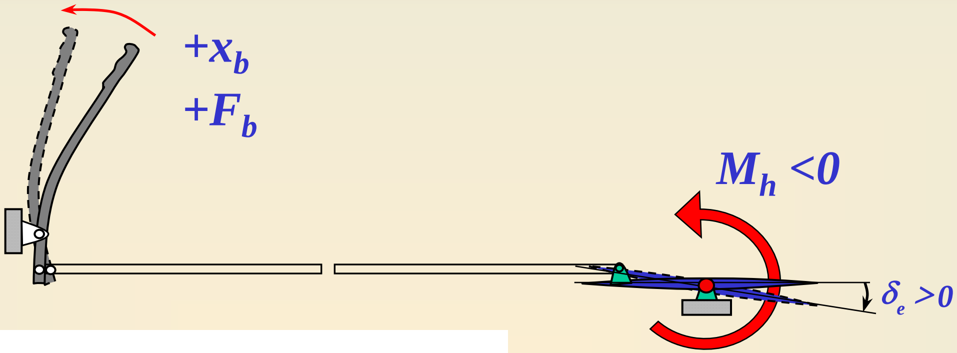
– decrease of control surface effectiveness:

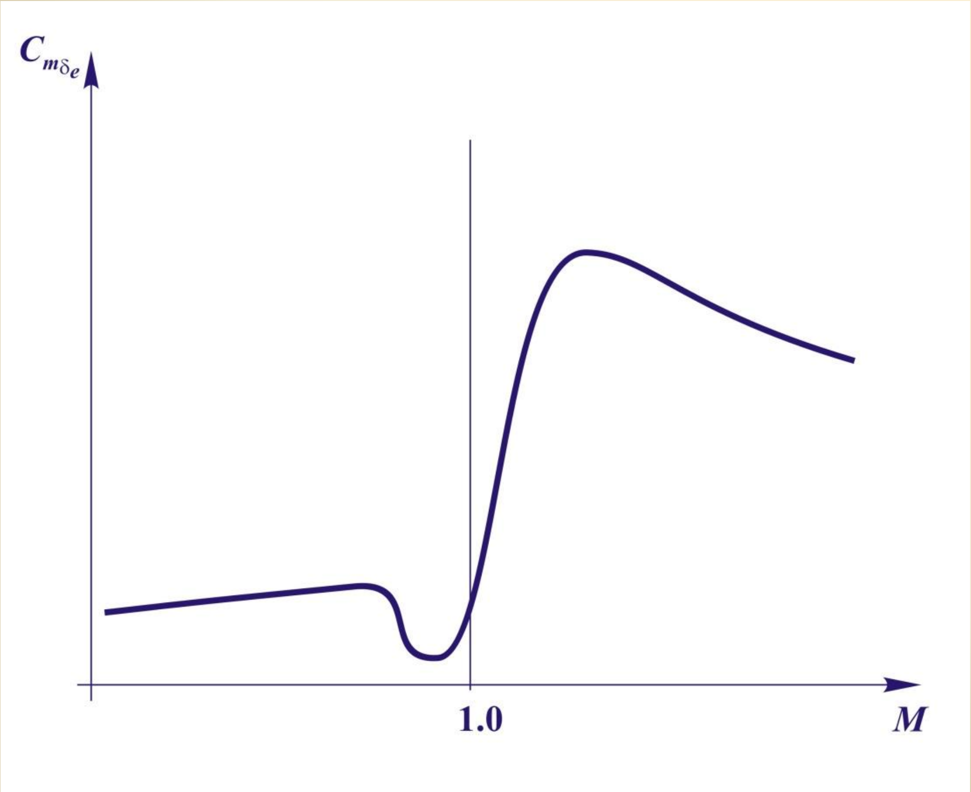


$$\Delta M = C_{m_{\delta_e}} \cdot \delta_e \cdot K \frac{\rho V^2}{2} S \cdot \bar{c}$$

Aerodynamic moments

– increase of hinge moment

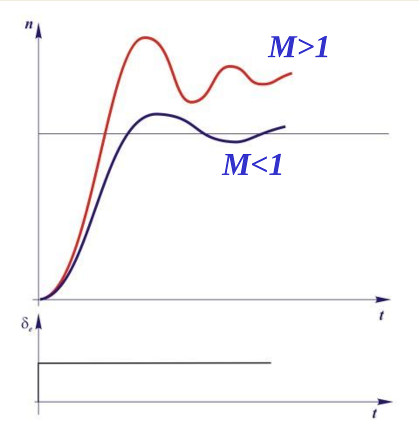




$$F_b = -\frac{\partial \delta_e}{\partial x_b} M_h$$

$$M_h = C_{m_h} \bar{c} \frac{\rho V^2}{2} S$$

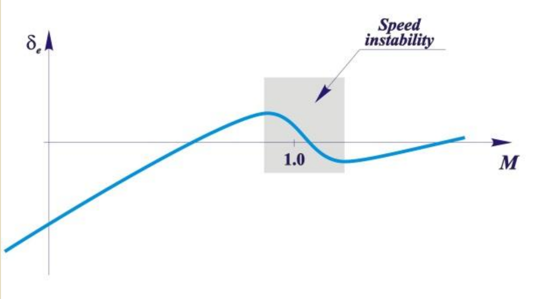
Consequences: considerable deterioration of flying qualities



– considerable change of dynamic responses

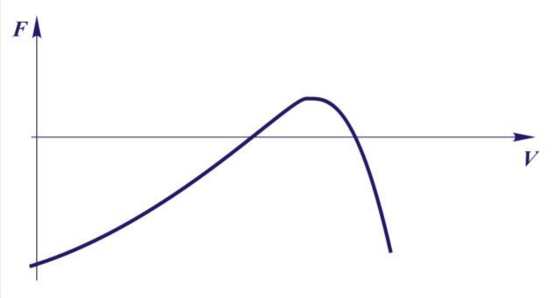
Controls provided horizontal flight

$$m_{z_0} + m_z^{C_y} C_y + m_z^{\delta}\delta = 0$$



δ_e – elevator deflection

$$\delta = \frac{m_{z_0} + m_z^{C_y} C_y}{m_z^{\delta}}$$

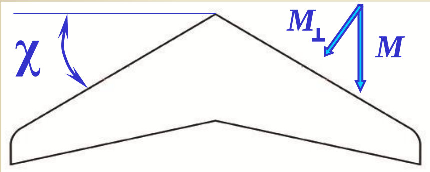


– considerable increase of forces (*F*) in transonic region

Conclusion: stability – did not guarantee necessary controllability

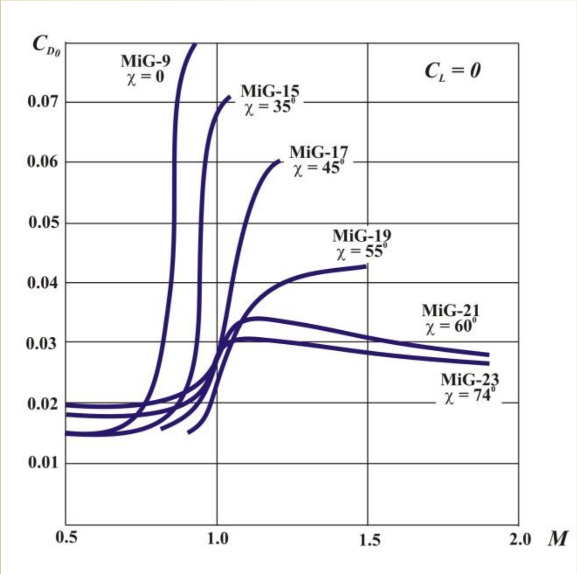
Solution of problems

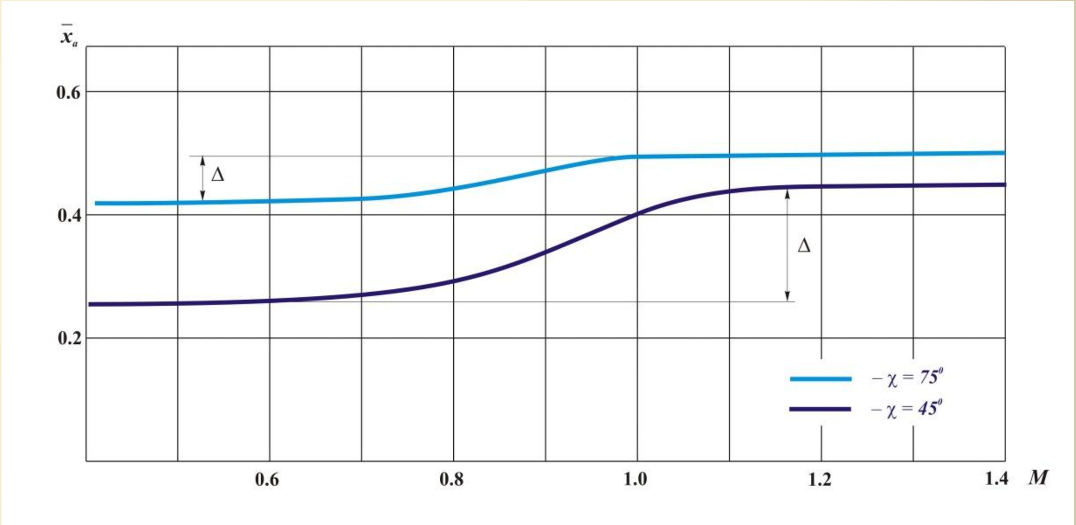
Increase of wing sweep χ



$$M_{\perp} = M \cdot Cos\ \chi$$

Effect:

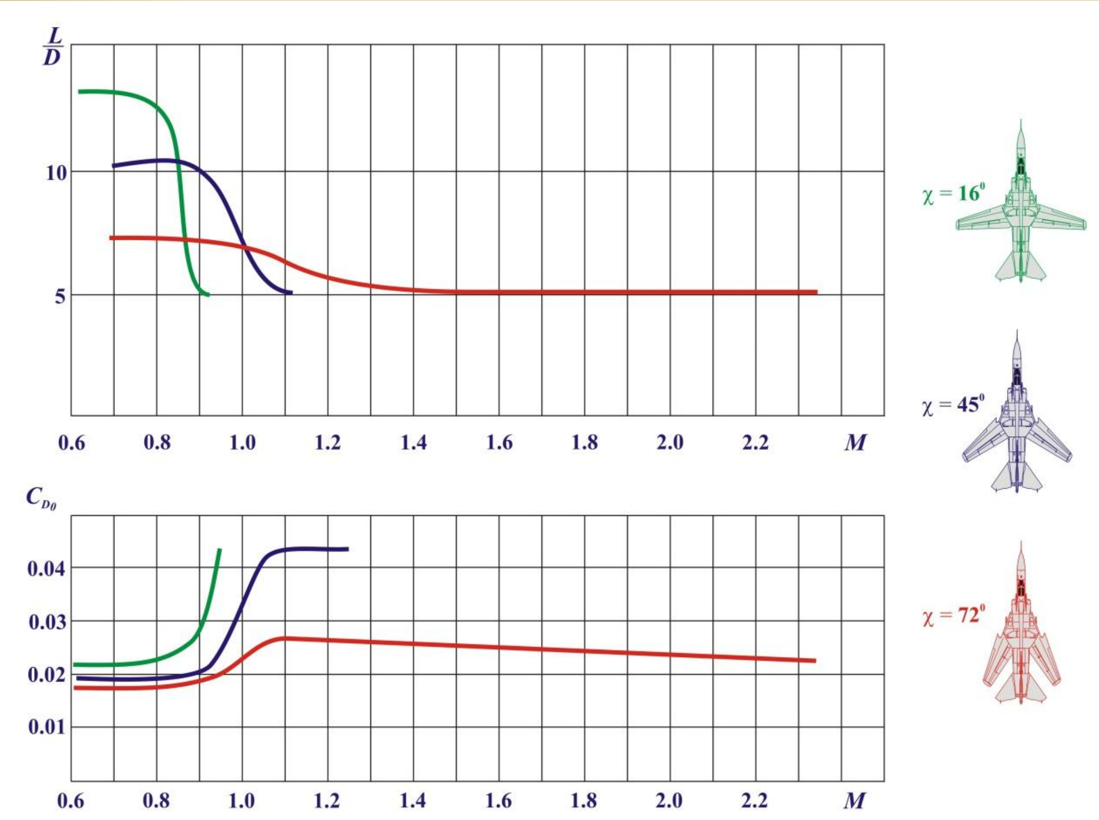




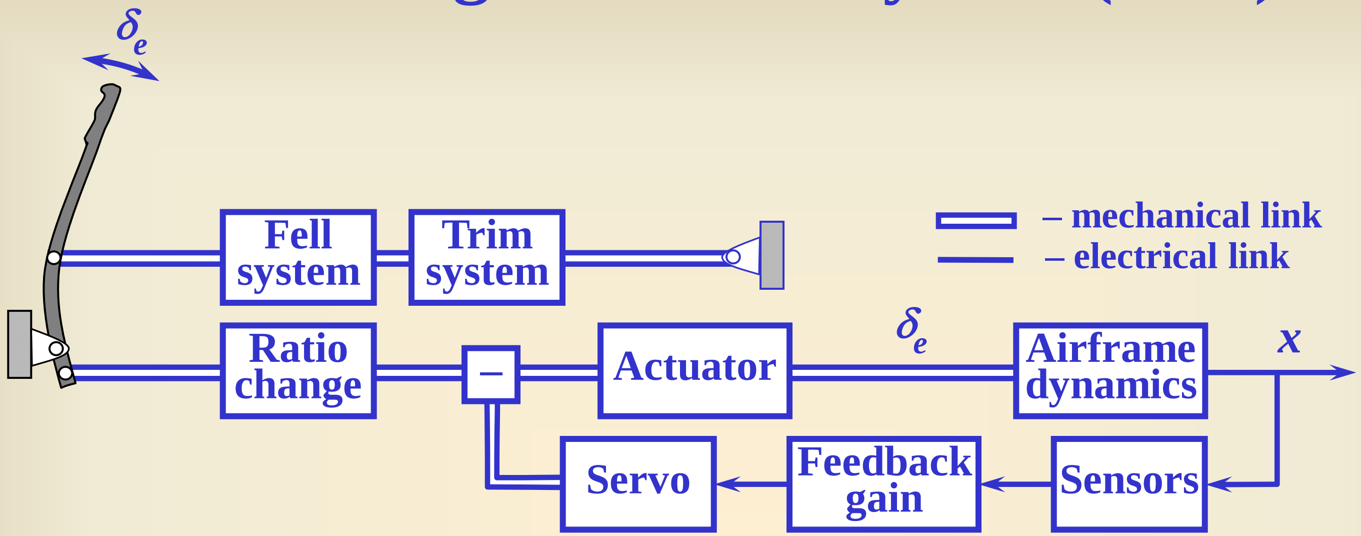
Several stages of wing sweep changes:

– sweep wing with subsonic leading edge MiG–21 (χ = 60⁰), thickness 4 ÷ 5%
– sweep wing with supersonic leading edge MiG–25 (χ = 40⁰), thickness 3 ÷ 5%
– sweep wing with variable sweep MiG–23 (χ = 16⁰ ÷ 74⁰), thickness 3 ÷ 4%

Aircraft with variable of wing sweep



Use of Flight Control System (FCS)



Typical features in FCS design:

– mechanical stick-to-elevator link

– aircraft is stable statically in longitudinal motion

– FCS influences on poles

$D(s)$ only:

$$W_c = \frac{x(s)}{\delta_e(s)} = \frac{N(s)}{D(s)}$$

Use of flight control systems and actuators

Advanced control law for passenger aircraft Tu-154

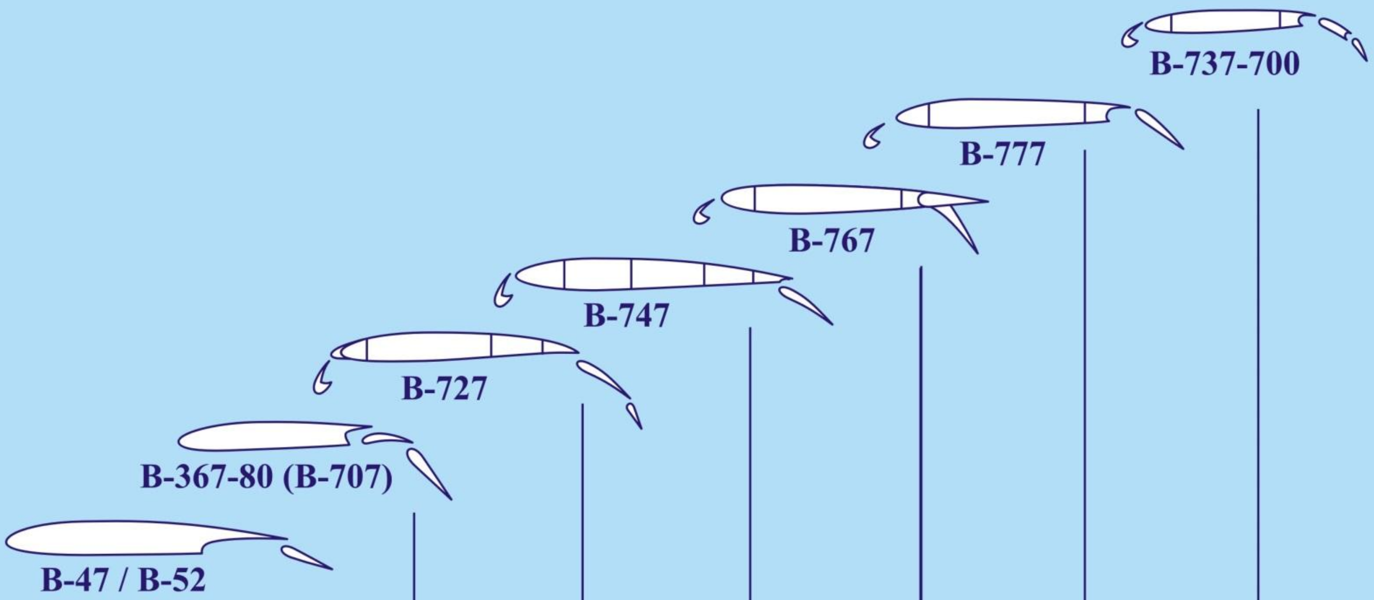
$$\delta_e = f(n_z, q)$$

Tu-154

first flight – 1968

Tu-154 is the first passenger aircraft with triplex redundancy of hydraulic system

Use of wing high–lift devices



	B-47 / B-52	B-367-80 (B-707)	B-727	B-747	B-767	B-777	B-737-700
$C_{L_{max}}$ landing	1.8	1.78	2.79	2.55	2.45	2.57	2.78
$\frac{L}{D}_{max}$ take off	10.5	9.7	10.0	10.0	12.6	12.6	11.5

Current challenges and potentialities

Human factor is a main **source (70÷80%) of accidents in aviation**

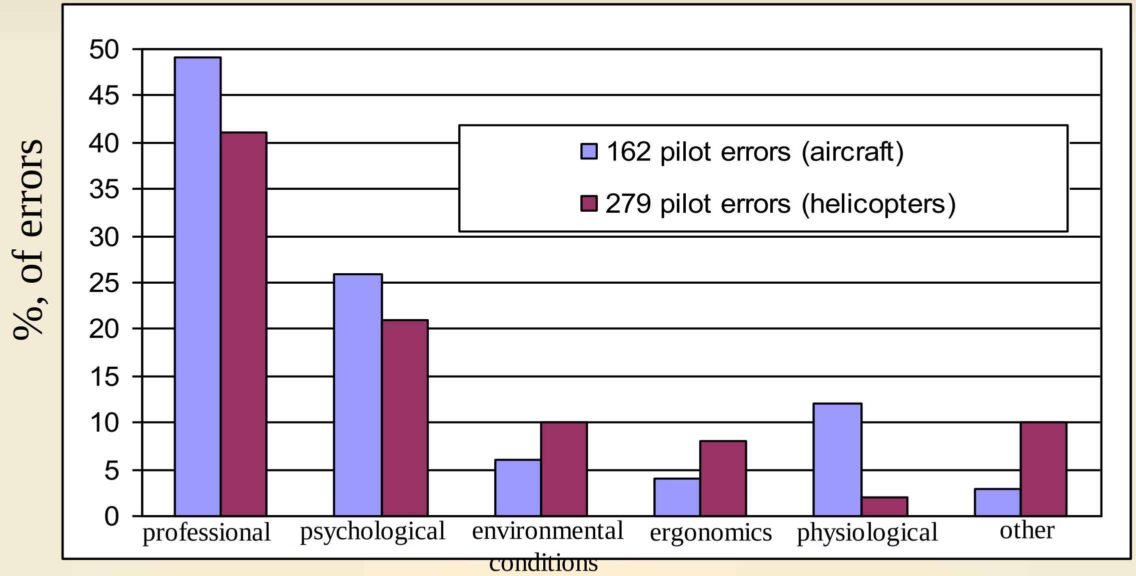
This factor was not taking into account in design many years before.

Human factors are defined by:

-Pilot's errors 71 – 75 %

- Other reasons (low professional level of technical maintenance, airport service, ATC, etc)

Components of pilot's errors



Professional – pilot wrong actions in case of failure, critical regime, etc.

Psychological – low stability to stress, impossibility to predict the situation.

Environmental condition – wind shears, fog ….

Ergonomics – unsatisfactory location of instruments, manipulators, brightness of symbols …

Psychophysiological – collusions, action of acceleration lower then threshold, very limited time margin for recognition of failure.

The ways for reduction of pilot's errors:

- Pilot training
- Aircraft system design provided necessary level of flight safety

The ways for the solution of problem:

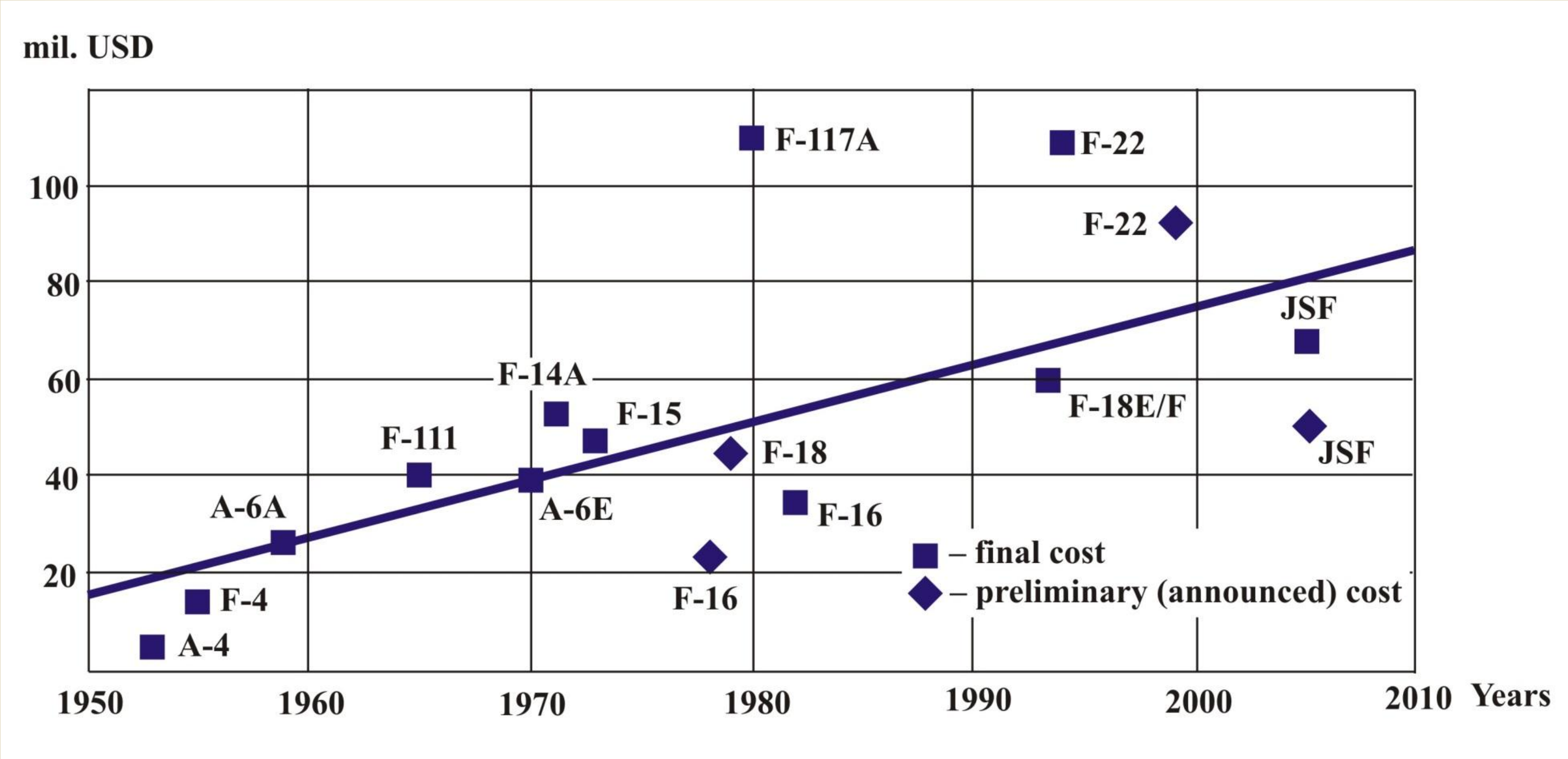
- To exclude the human operator from the control (autonomous control)

Role of operator is passive (monitoring, realization of supervisory control). He participates in control to change the program or regime in accident case or sudden change of environmental conditions.

- To participate in control activity (manual control), by the best integration of operator and machine.

Challenges for military aircraft

– **Effectiveness and cost**



– **Flight safety**

Challenges for civilian aircraft

(formulated by European council for aeronautical research)

- **quality and affordability;**
- **environmental preservation (halving full consumption per pax. *km*, cutting NO_x by 80%, reducing perceived external raise by 50%) ;**
- **safety: reduction of accident rate by 80% and reduction of human error in four times;**
- **efficiency: increase of the air transport system in terms of capacity to accommodate three times more aircraft movement in 2020 to ensure on time flights;**
- **security, the goal being Zero successful attack or hijack.**

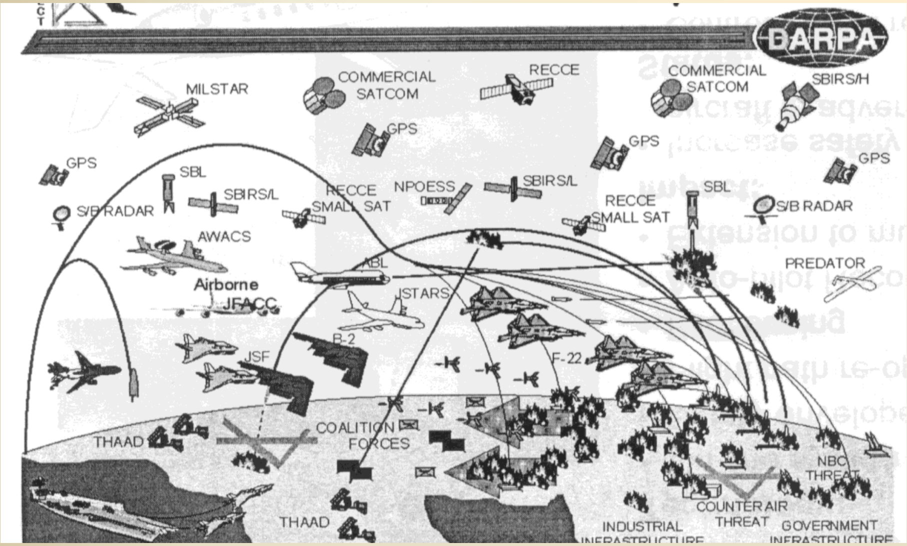
All these challenges can be achieved only by cosideration of a global system approach involving

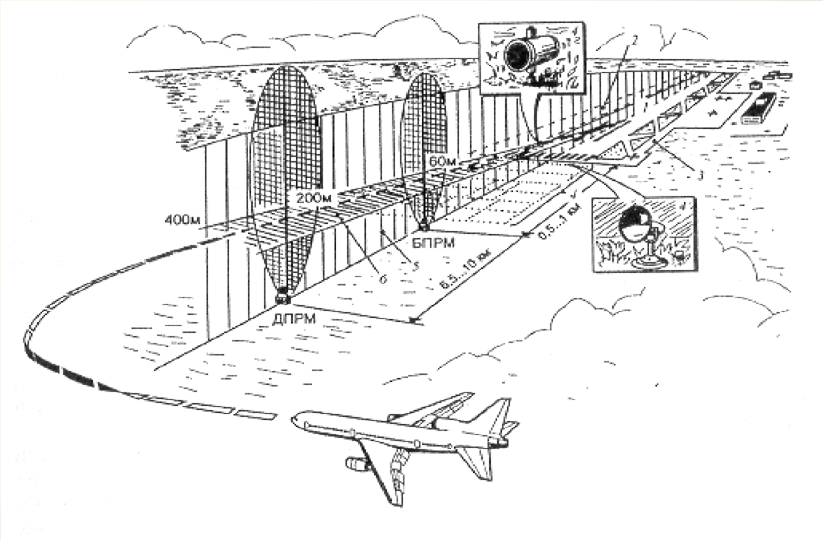
For military aviation:

- **aircraft;**
- **group of different vehicles (space, ground, enemy);**
- **management system**

For civilian aviation:

- **air transport system;**
- **airport;**
- **air traffic management;**
- **aircraft**





Technical and scientific potentialities:

– achievements in aerodynamics, materials, propulsion;
– multidisciplinary approach to design;
– reliability of subsystems (computers, actuators, …)

Challenges + potentialities

↓

New principles

– change of flight control system role and its technology;
– optimization of wing aerodynamics;
– super maneuverable flight with unlimited angles of attack;
– interface friendliness;
– unmanned air vehicles
– international cooperation in aero / astro area

Innovations based on new principles

Break through in aviation

***New principle*: Change of FCS role and its technology**

In the past – FCS system is a subsystem

– for:

a) realization of piloting task;

b) improvement of flying qualities;

– characterized by mechanical linkage between stick (wheel) and effectors (actuator)

Current – FCS system is a subsystem

– for:

a) improvement of flying performances;

b) provision of necessary flying qualities;

c) provision of necessary flight safety level;

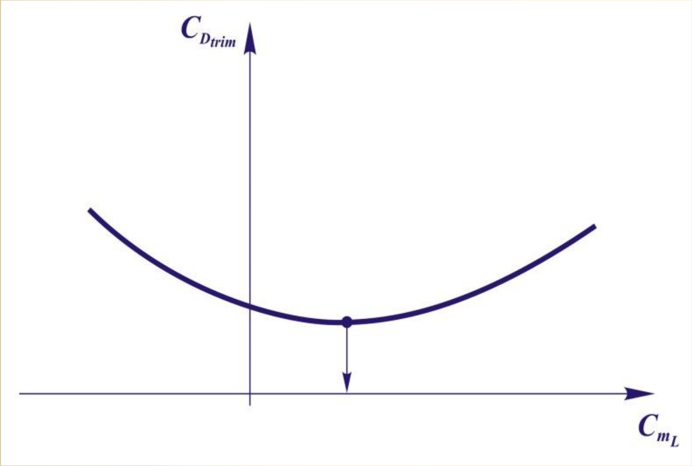
– characterized by electrical linkage between stick (wheel) and actuator (FBW technology)

New principle: Change of FCS role and its technology

a) FCS for improvement of flight performances

Innovations:

a.1) increase of instability and provision of controllability with FCS



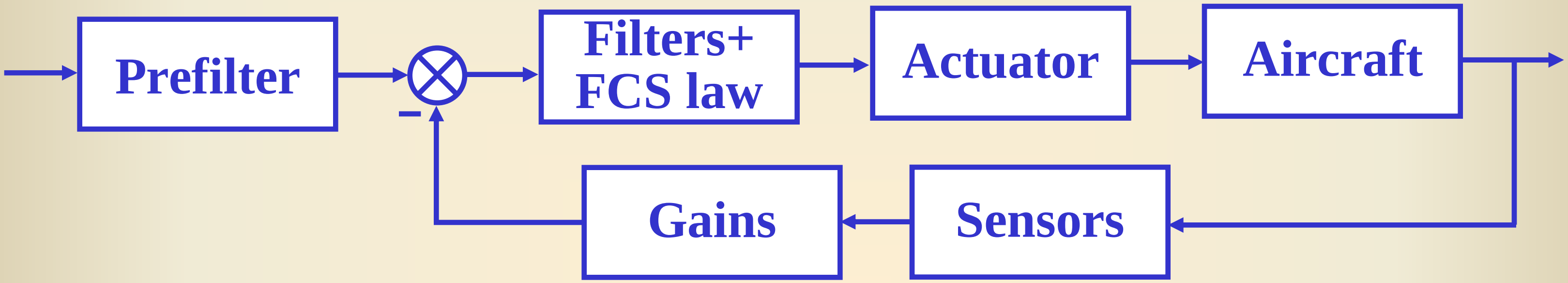
$C_{m_L} \approx 0 \div 5\%$

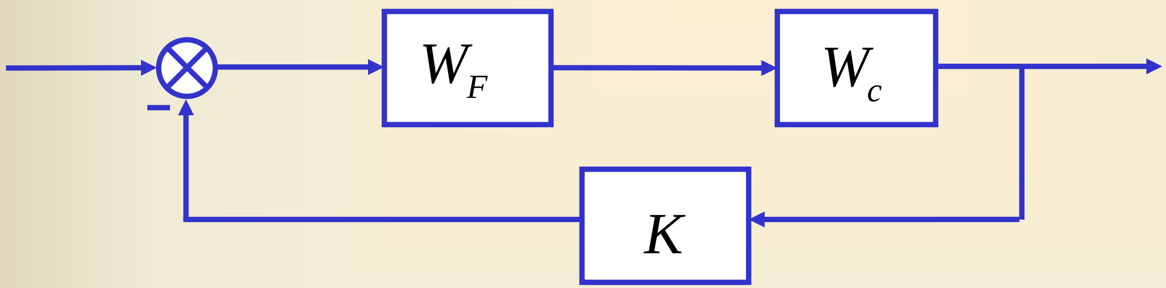
$C_{m_L} \approx 10 \div 15\%$

$C_{m_L} \approx 20 \div 25\%$

Особенности динамики высокоавтоматизированных ЛА

Обеспечение необходимой устойчивости и управляемости с помощью системы управления

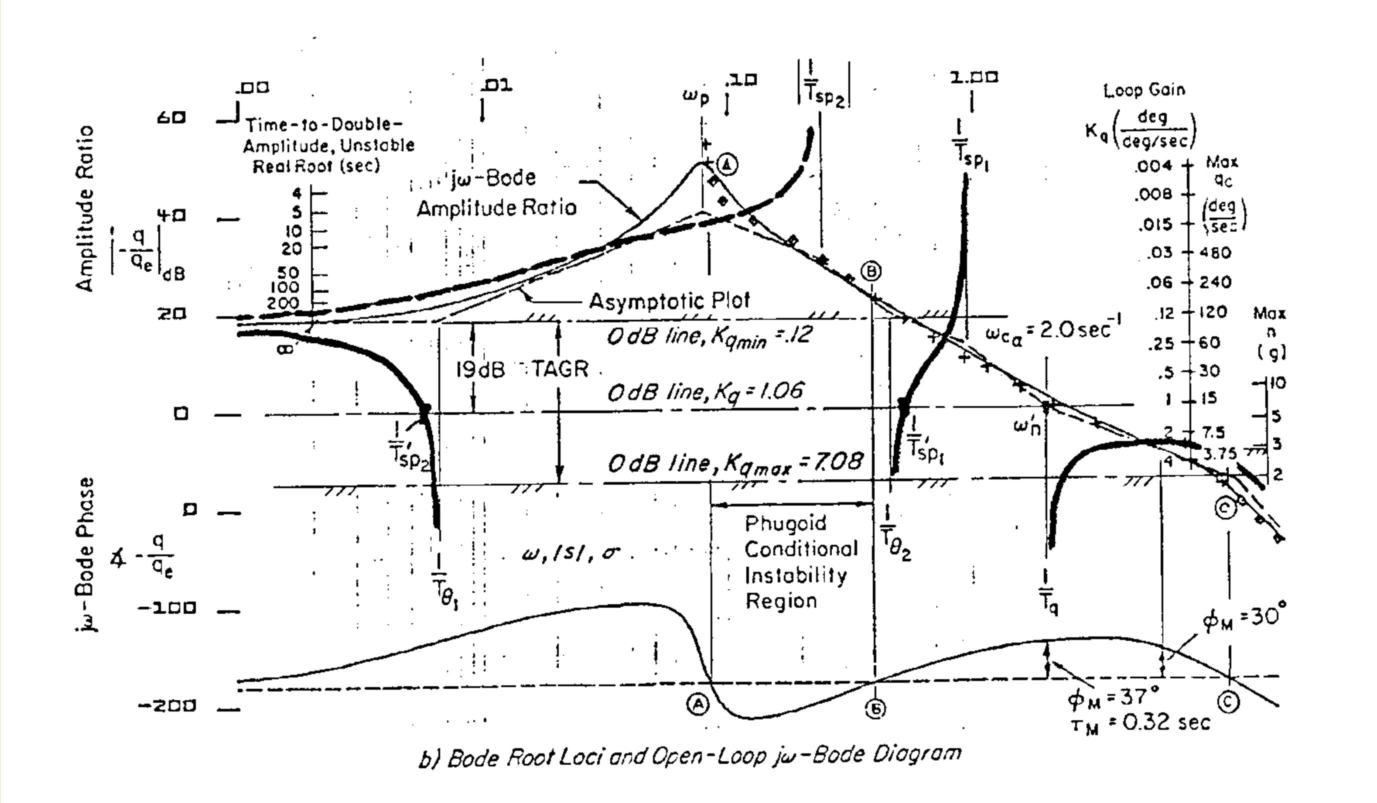




$$\bar{W}_c = \frac{W_F W_c}{1 + W_F W_c K}$$

Запаздывание в тракте управления

$$\bar{W}_c = W_c^* e^{-p\tau}$$



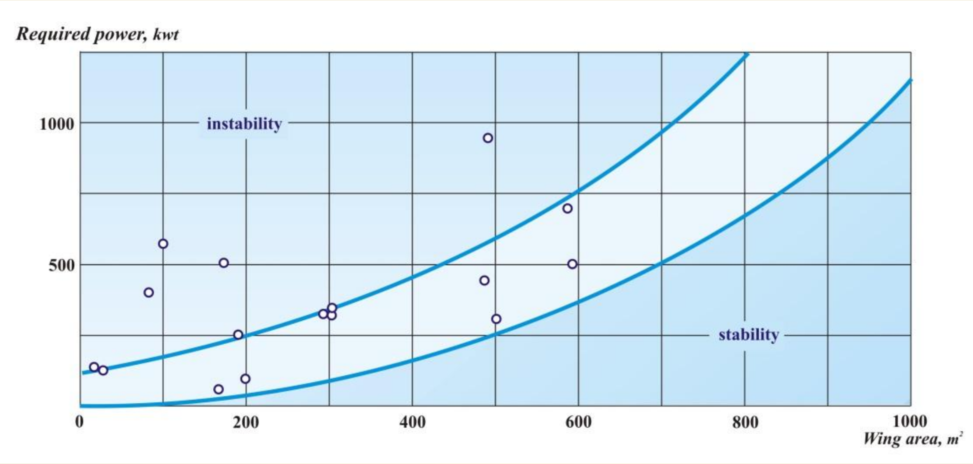
b) Bode Root Loci and Open-Loop jω-Bode Diagram

Неустойчивость в длиннопериодическом движении

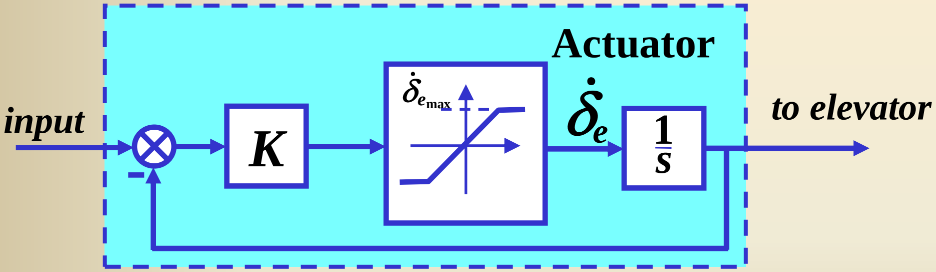
$$\omega_d^2 = g\left(\frac{-\bar{Y}^V \bar{M}_z^\alpha + \bar{Y}^\alpha \bar{Z}^V}{\omega_k^2}\right)$$

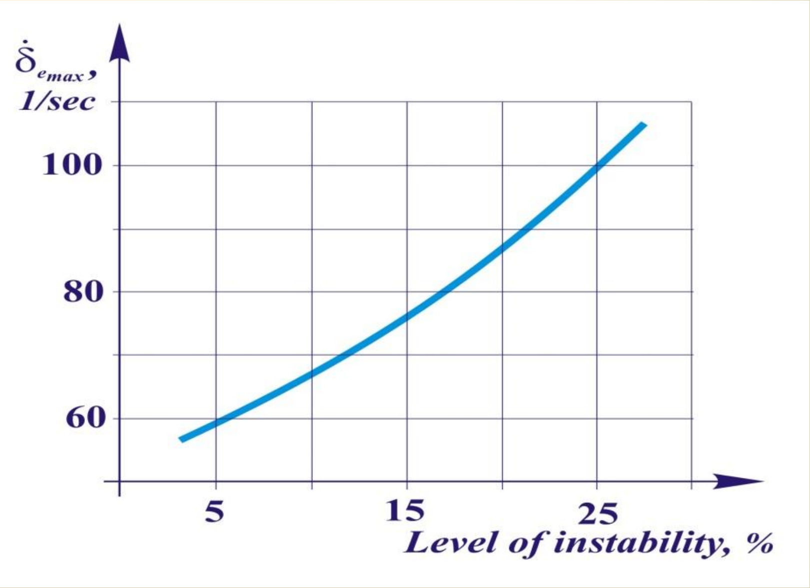
Повышение требований к приводам

– increase of required power for unstable aircraft

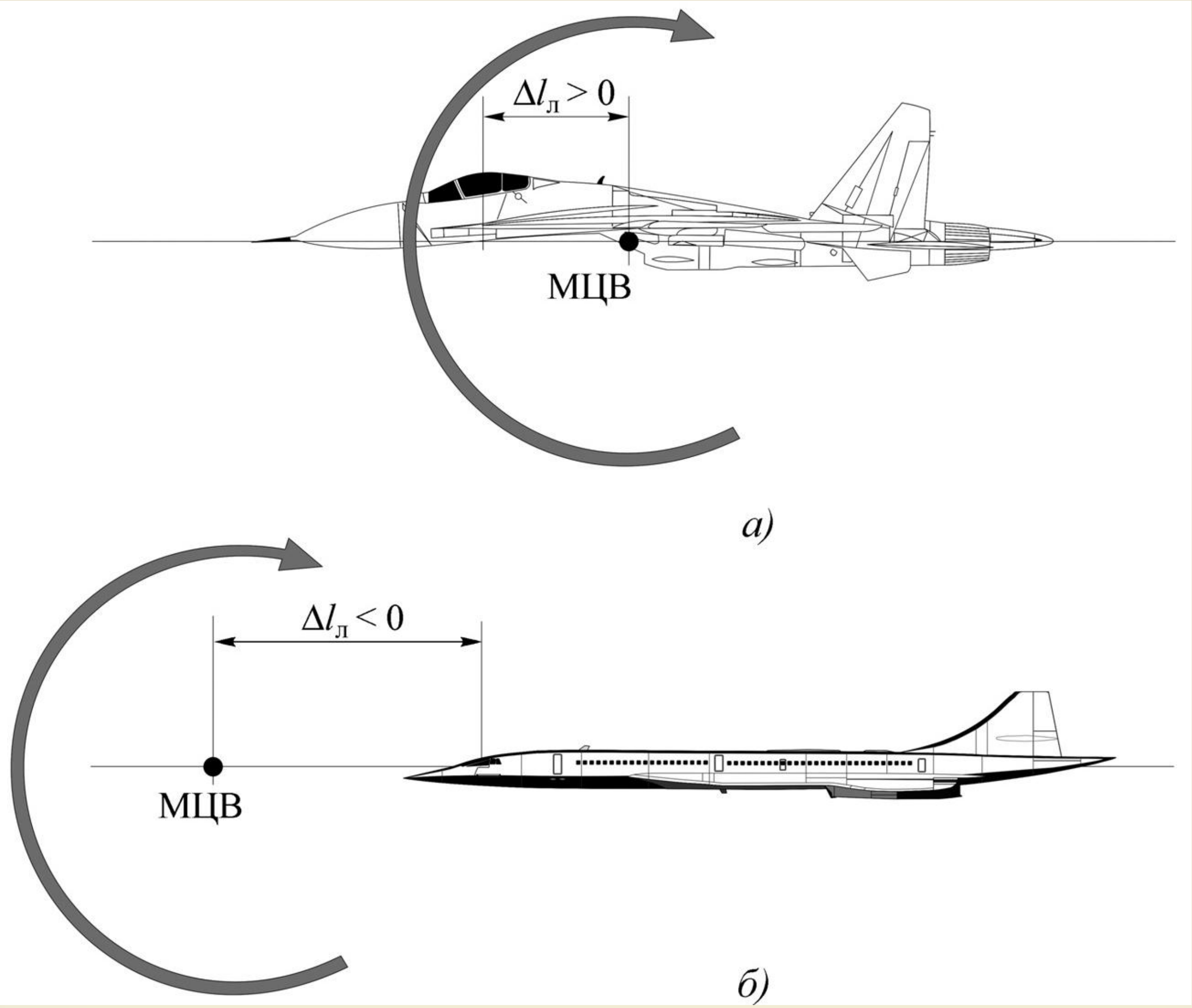


– increase of required rate limit





Особенности схем некоторых современных самолетов



$$\Delta l_л = l_л + \frac{\bar{Y}^{\delta_в} V}{\bar{M}_z^{\delta_в}}$$

a.3.1. Transformation of aircraft to "more electrical aircraft"

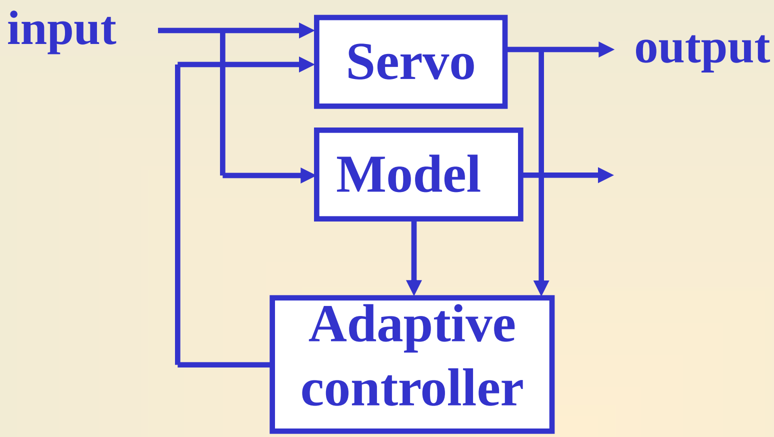
Transformation of actuators

Hydro–mechanical → Electro hydraulic → Electrical

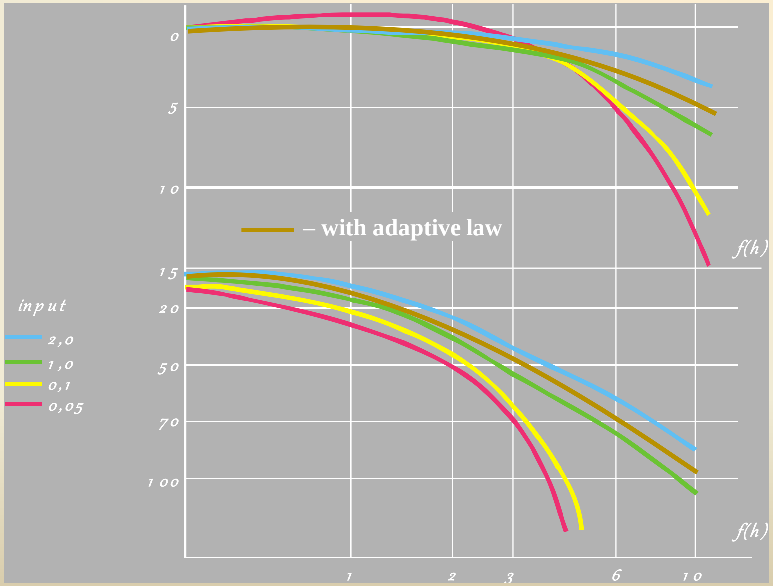
1950 → 1970 → 2005 ÷ 2010

a.3.2. Actuator with adaptive law

Self-adaptation of actuator



Actuator frequency response



-Improvement of frequency characteristics

-Reduction of requirements to rate limit, $\dot{\delta}_{max}$

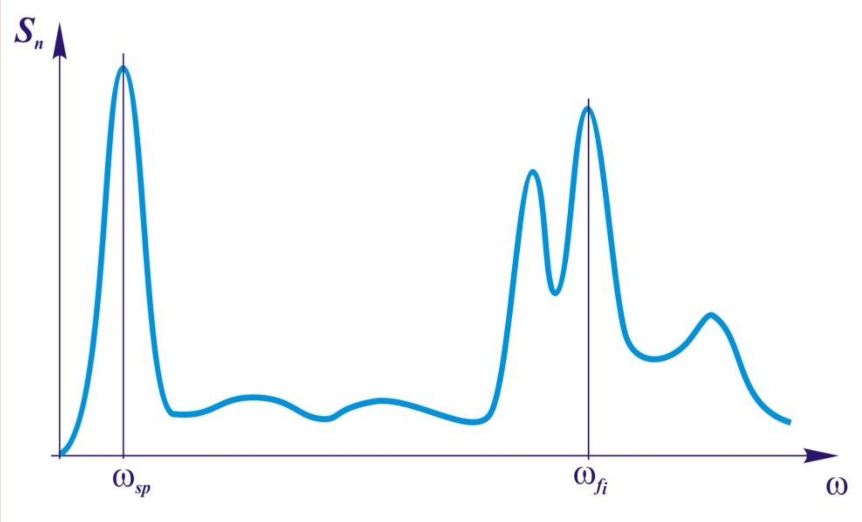
a.4. Active Flight Control System (AFCS) for:

– suppression of flexibility and atmosphere turbulence;

– decrease of bend moments on wing in maneuvers

Necessity of such system:

a)



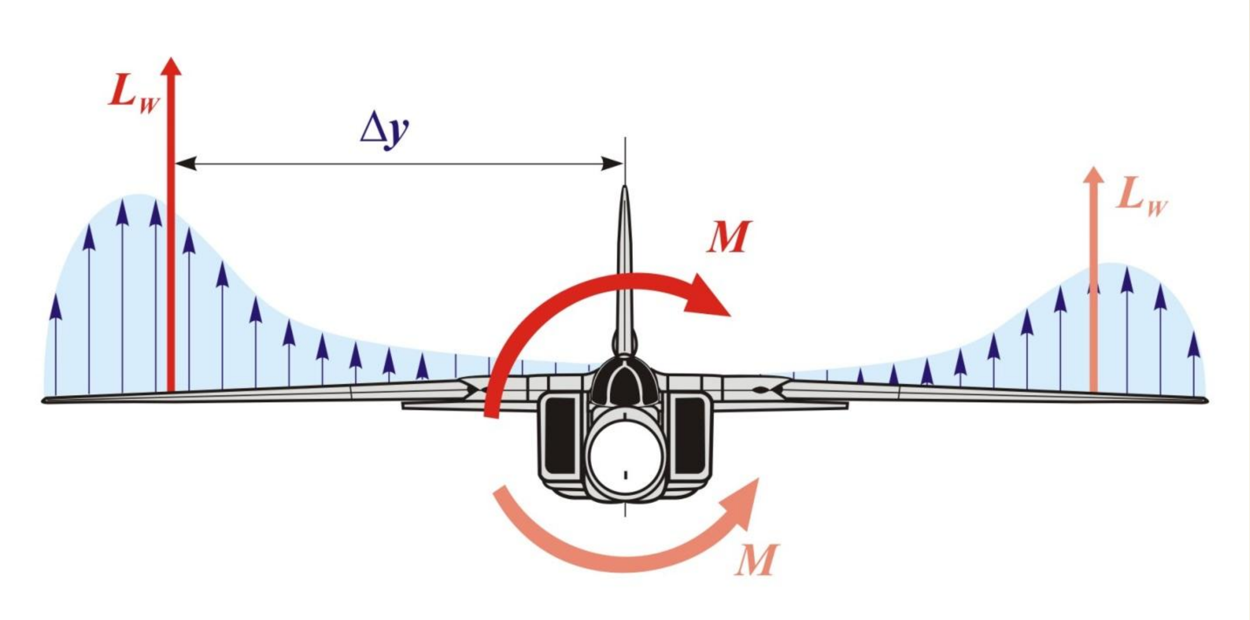
ω_{f_i} **– frequencies of oscillation modes of flexible structure**

ω_{sp} **– short period frequency**

	ω_{f_i} , 1/sec	ω_{sp} , 1/sec
In the past	30 ÷ 30	2 ÷ 3
now	5 ÷ 10	2 ÷ 3

$\omega_{f_i} \longrightarrow \omega_{sp}$

b)



$$M = L_W \cdot \Delta y$$

In maneuver $\Delta y \uparrow$, $L_W \uparrow \;\rightarrow\; M \uparrow$

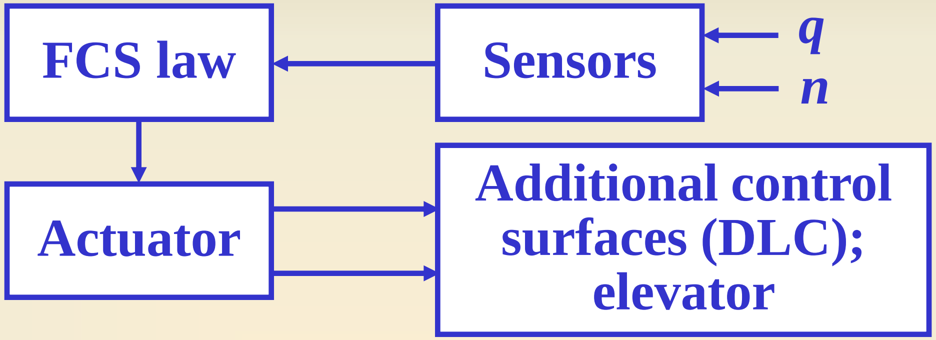
Consequence:

– decrease of service life;

– discomfort;

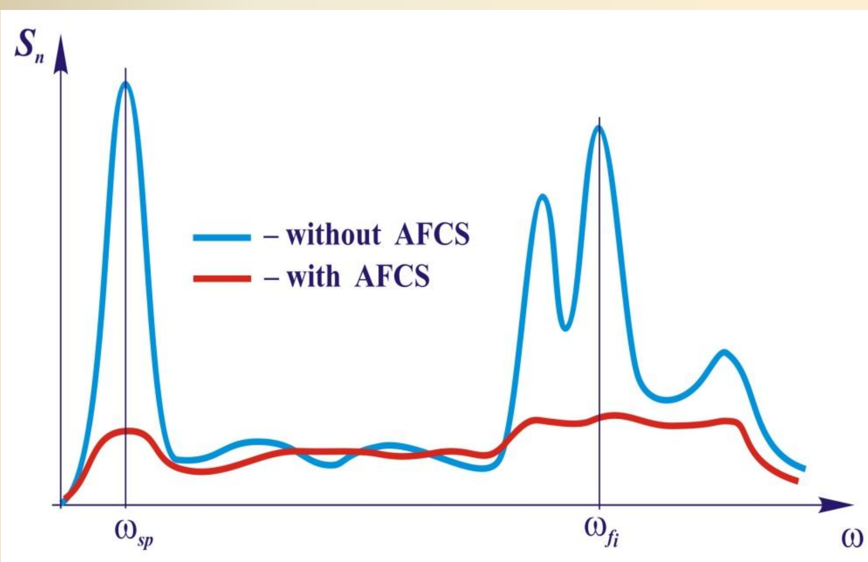
– increase of probability of flutter

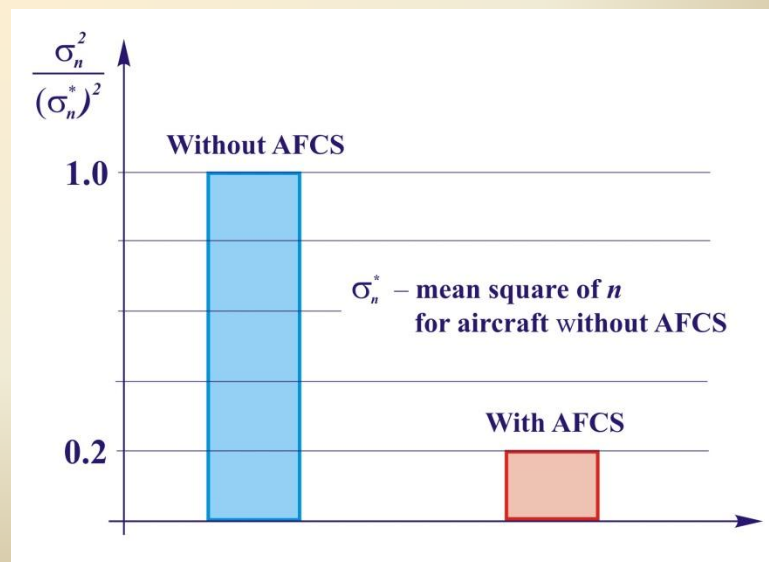
Structure of active FCS



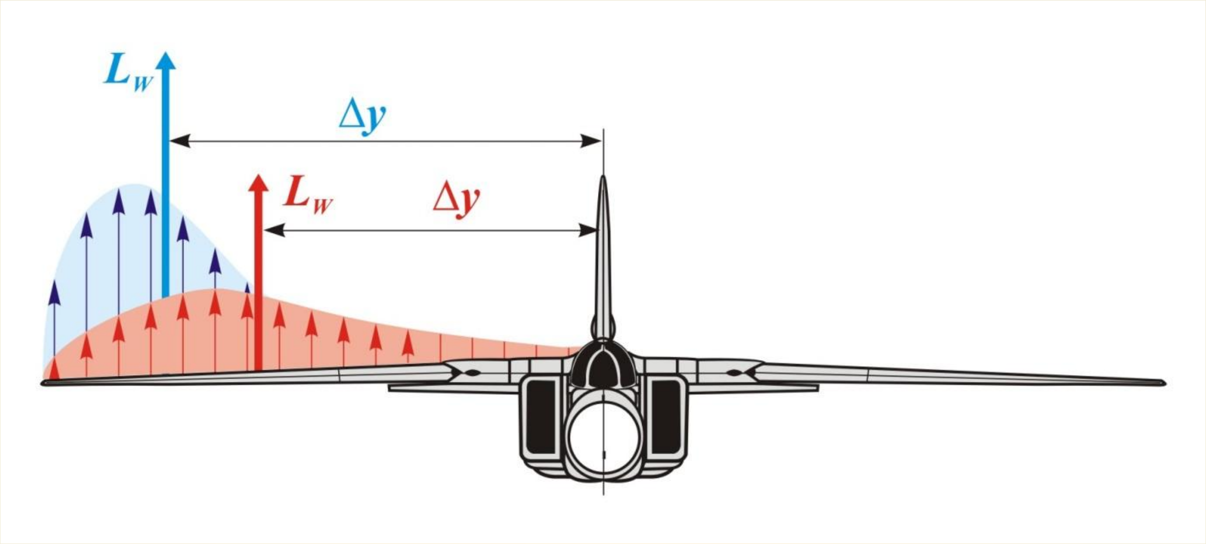
Effect of active FCS

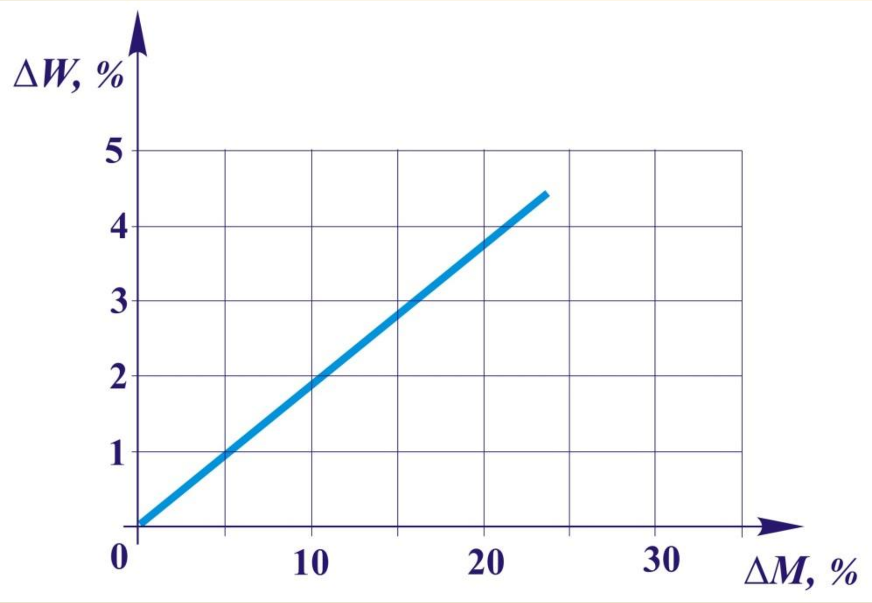
a) Decrease of effects of flexibility and atmosphere turbulence





b) Decrease of bending moment





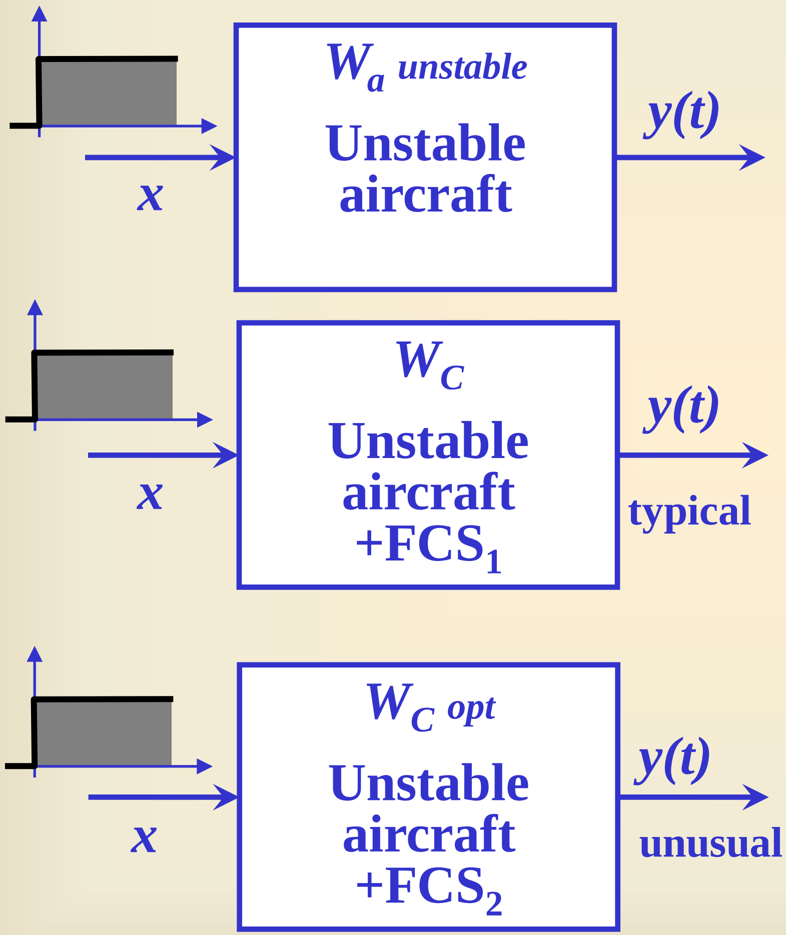
ΔM **– decrease of bending moment**

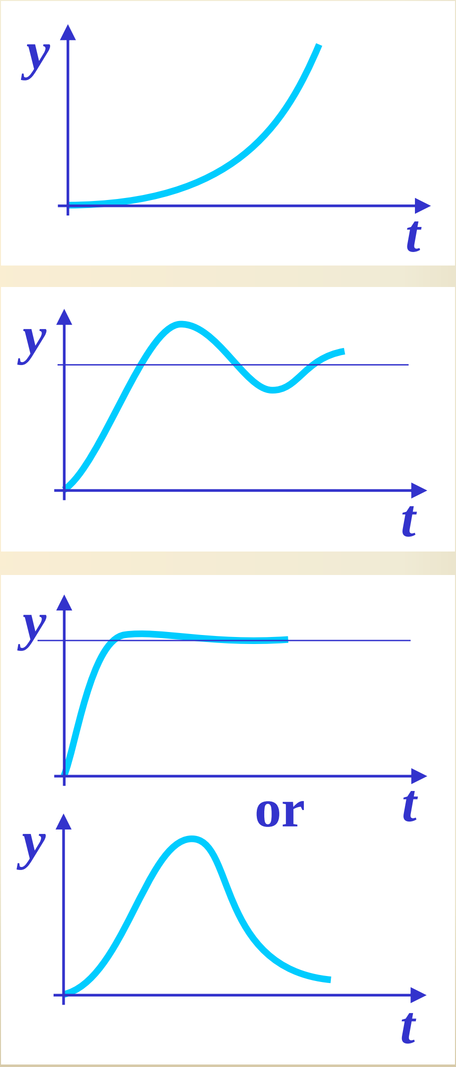
ΔW **– increase of weight**

New principle: Change of FCS role and its technology

b) FCS for provision of necessary flying qualities

HA FCS is able to provide any dynamic response

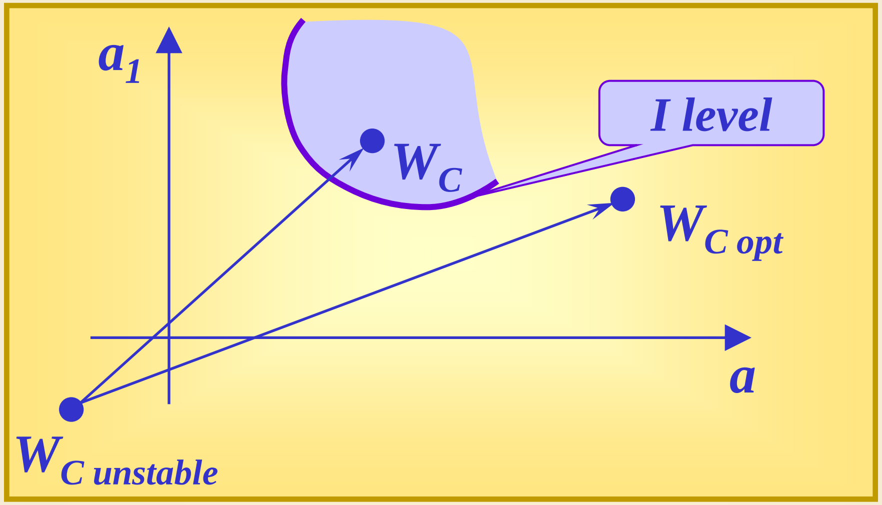




What type of response is necessary?

New approach

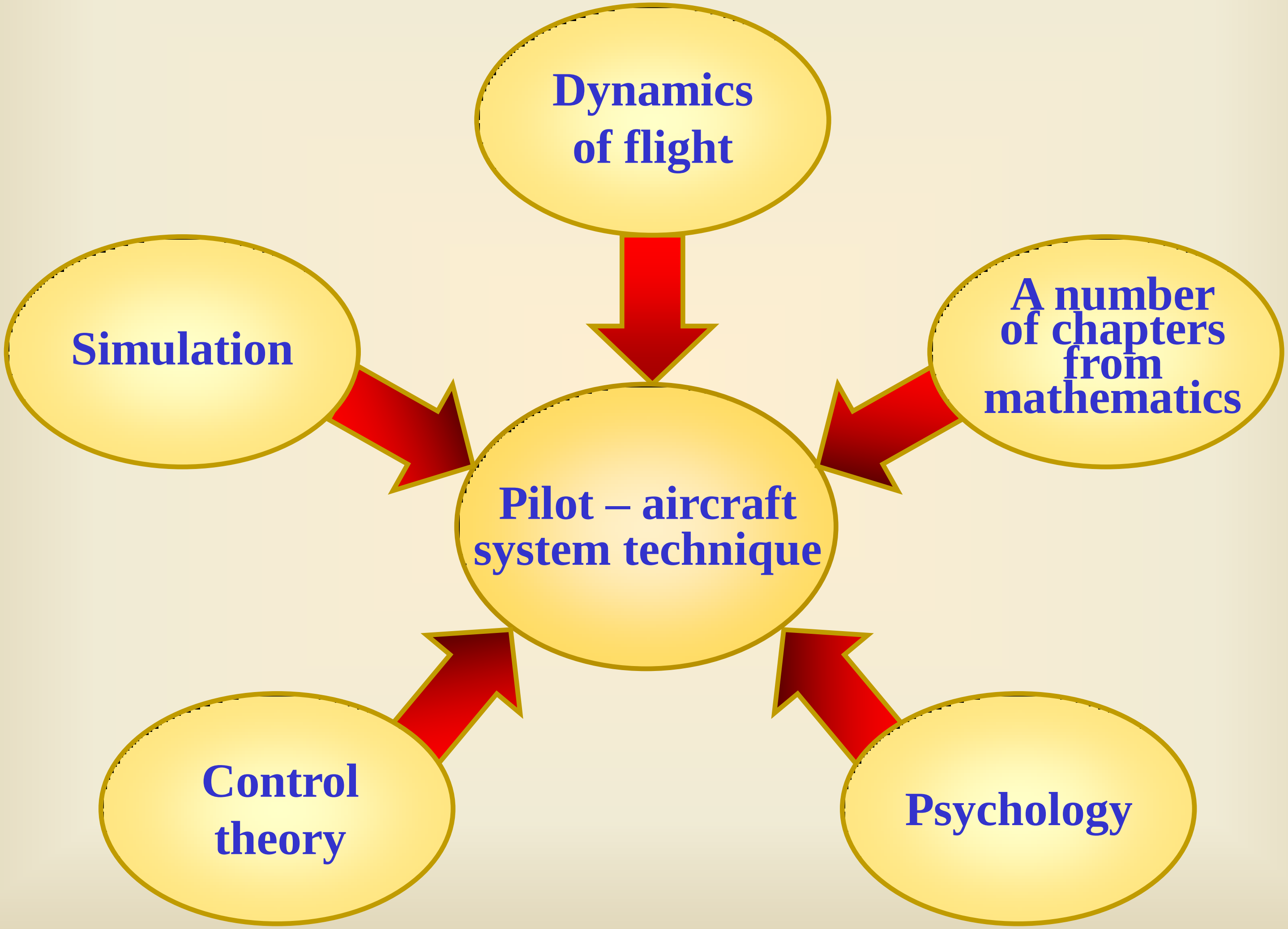
FQ has to correspond to optimal FQ ($W_{C\ opt}$) in each piloting task



Questions:

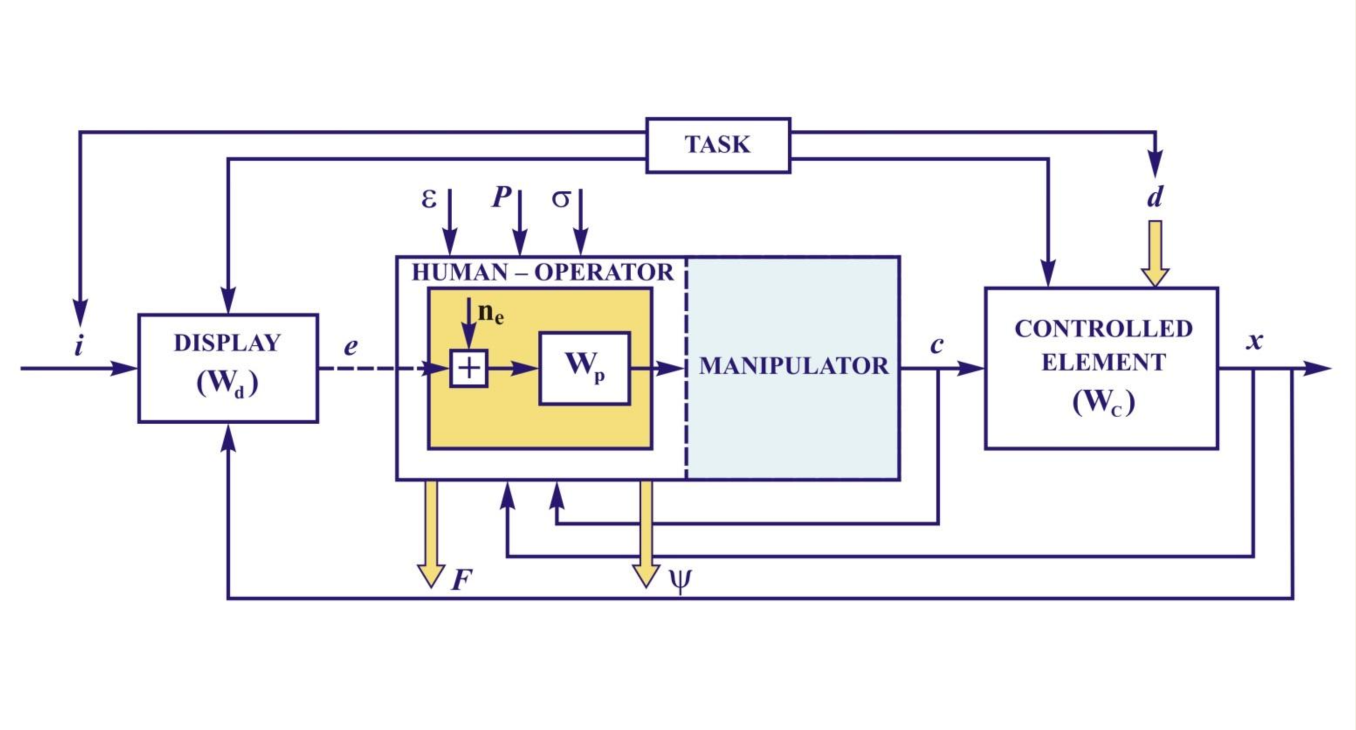
- **What does it mean optimal flying qualities?**
- **How piloting task influences on $W_{C\ opt}$?**

Innovation: Pilot – aircraft system technique



Pilot–vehicle system (PVS) peculiarities

1. Pilot and aircraft interaction takes place in closed–loop system



2. Specific feature of pilot-vehicle close-loop system is the <u>influence of the piloting task on all its elements</u> (task variables)

PVS technique:

– techniques for ground and in–flight experiments;

– algorithms for mathematical modeling of pilot response characteristics;

– software for simulator’s computers and data reduction system;

– equipment: simulators, workstation, different devices for investigations;

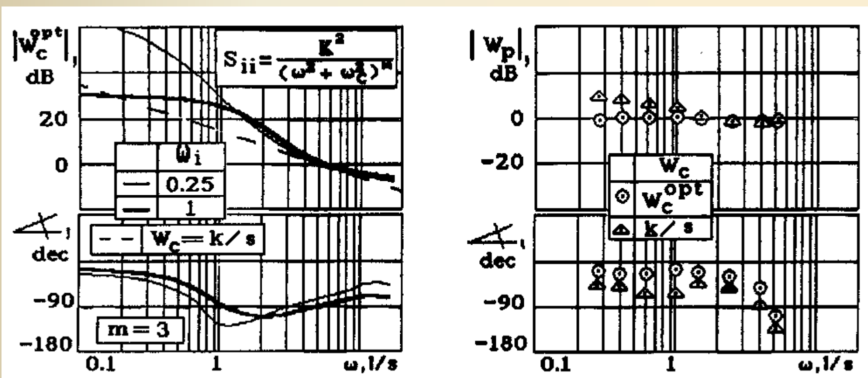
– data base on regularities of pilot behavior.

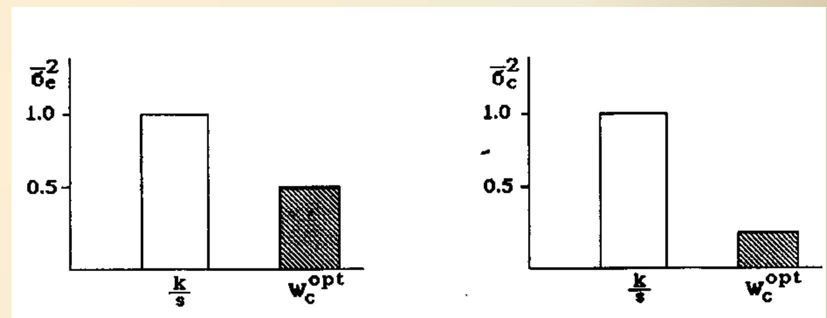
Application of PVS technique

1. Optimization of aircraft dynamics

task → **Technique for definition of $W_{C\ opt}$** ← Pilot's limitations (PL)

↓ $W_{C\ opt}(task, PL)$





2. Application to FCS design

– optimization of aircraft dynamics in aim–to–aim tracking task,
– FCS design of aircraft with DLC,
– compensation of time in pitch tracking task,
– FCS design in refueling task,
– unification of automatic and manual FCS in aim–to–aim tracking task.

New principle: Change of FCS role and its technology

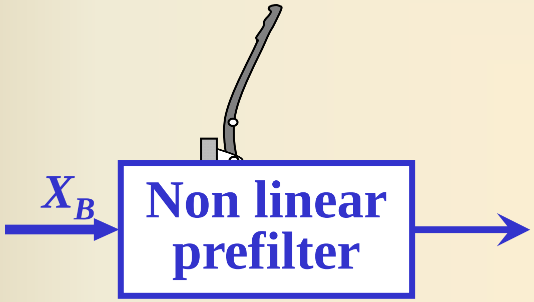
c) Provision of necessary flight safety level

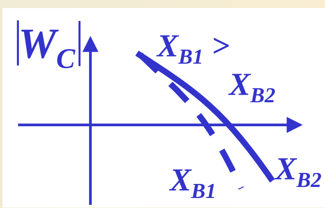
Factors influenced on controlled element dynamics:

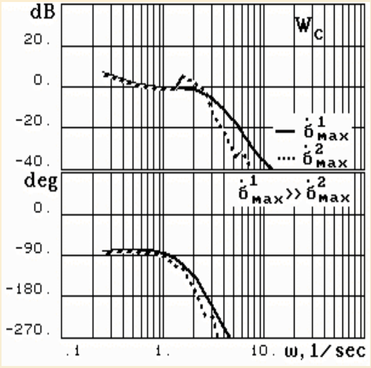
a) variability of *H*,*V* and aircraft parameters;

<u>Consequence</u>: non optimal aircraft dynamics

b) variability of nonlinear FCS dynamics for different input signal

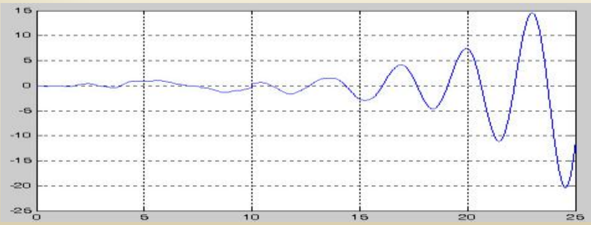






<u>Consequence</u>: deterioration of flying qualities

c) failure of hydraulic station

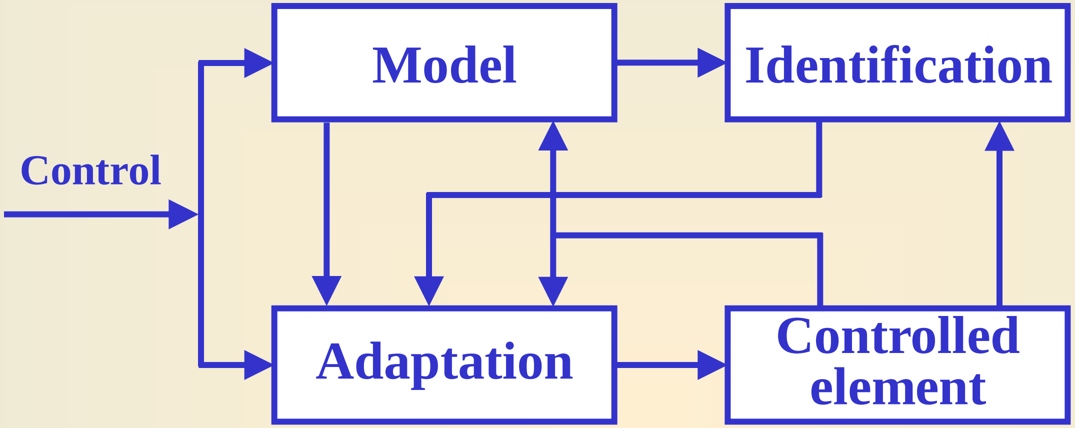


<u>Consequence</u>:

satisfactory FQ ➡ unsatisfactory FQ

Innovations:

c.1. FCS with adaptive law



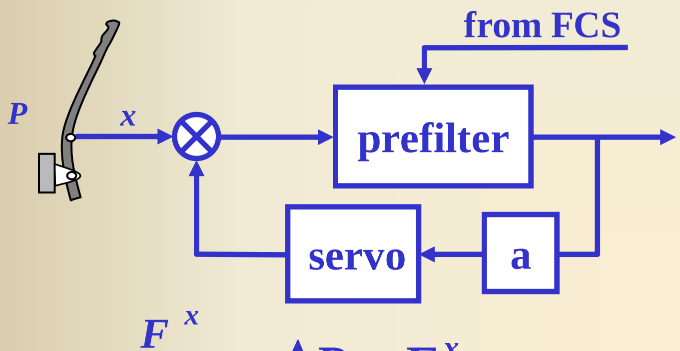
– Improvement of Flying Qualities

– Suppression of variability in aircraft parameters and failure effects

– Self adaptation is a mean for reduction of required $\dot{\delta}_{e\ max}$ (in 1.5÷2 times)

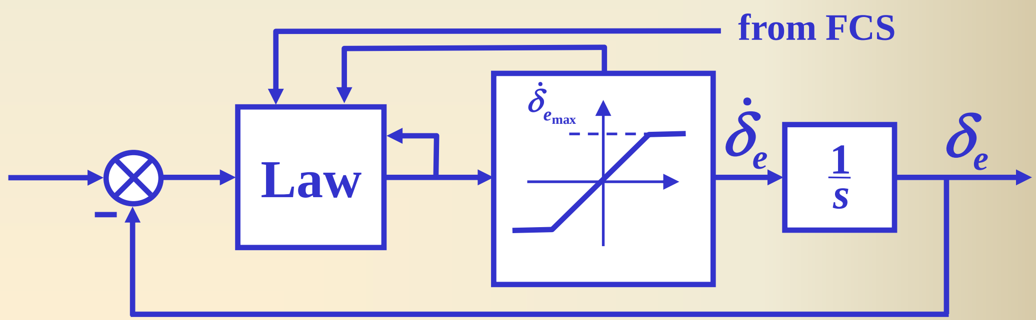
c.2. Means for conjunction of pilot action and FCS potentialities

The manipulator with changeable spring stiffness



$$\Delta P = F^{x} x_{np}$$

Adaptive prefilter

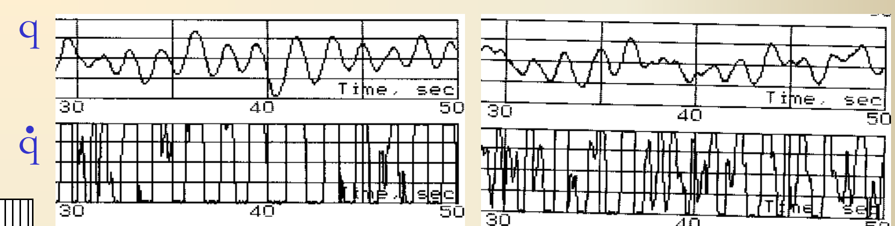


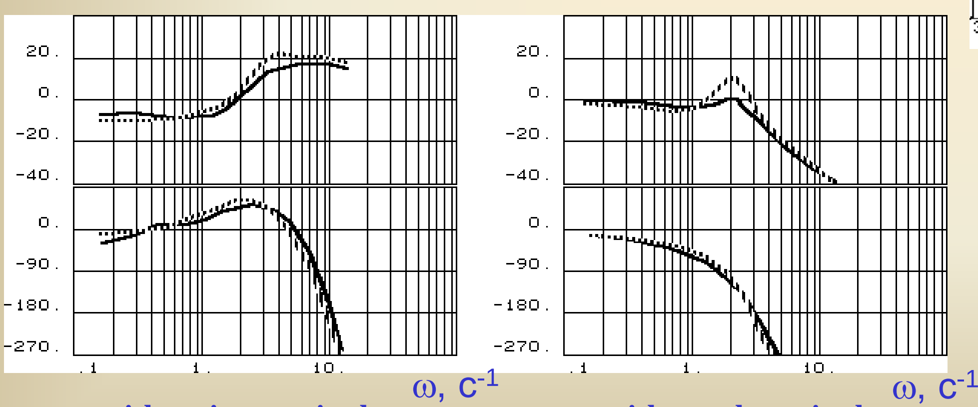
Linear model of prefilter

$$W_F = \frac{1}{T_f s + 1}$$

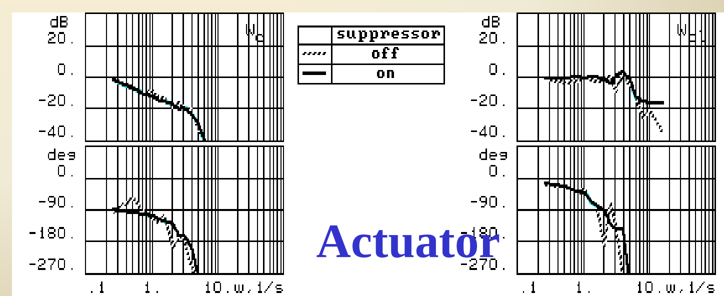
$$\left\{\frac{P}{x}\right\} = F^{x} \times (1-a) \times \frac{\frac{T_f}{1-a} s + 1}{T_f s + 1}$$

without suppressor | with suppressor



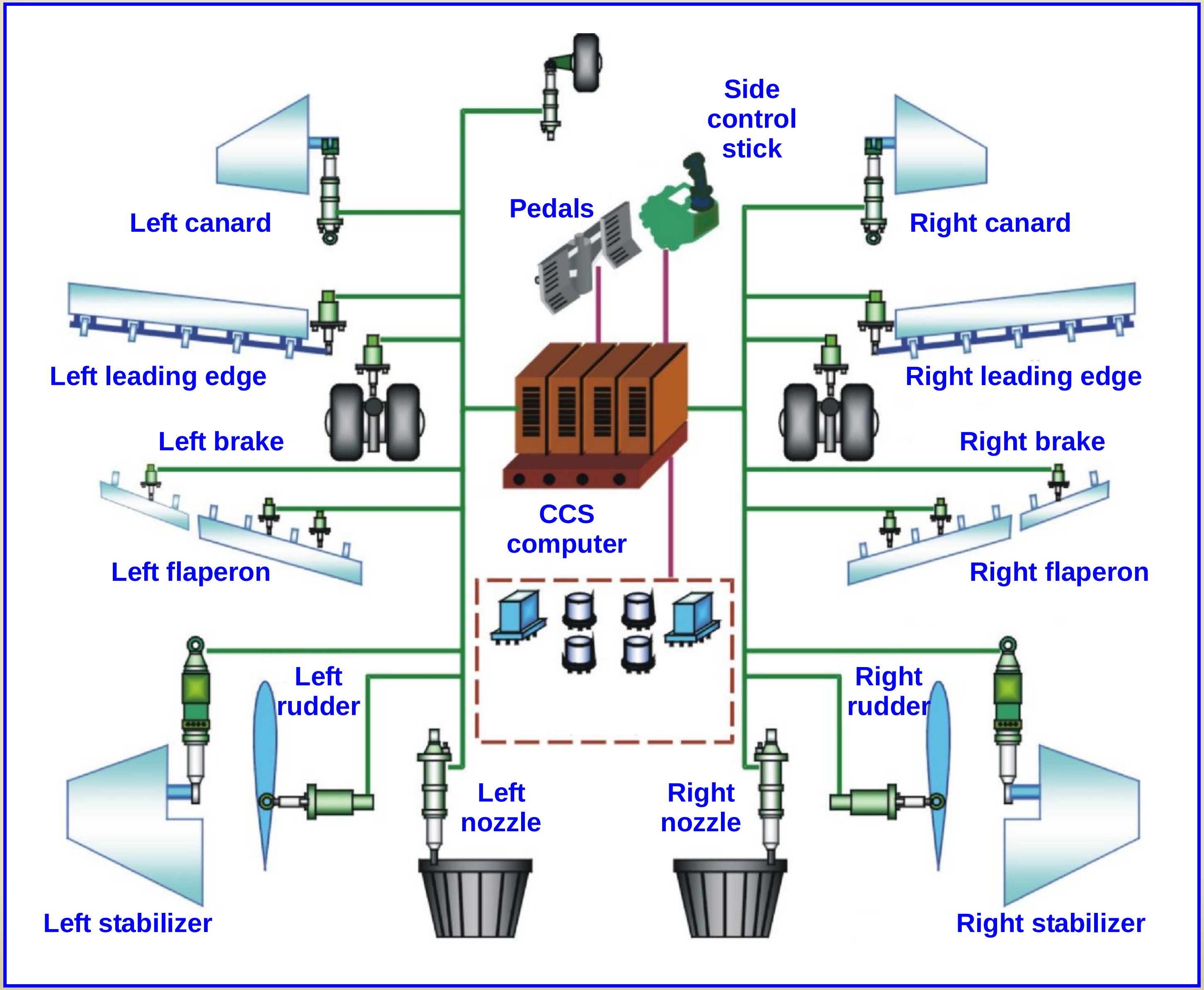


—— with active manipulator .... with usual manipulator



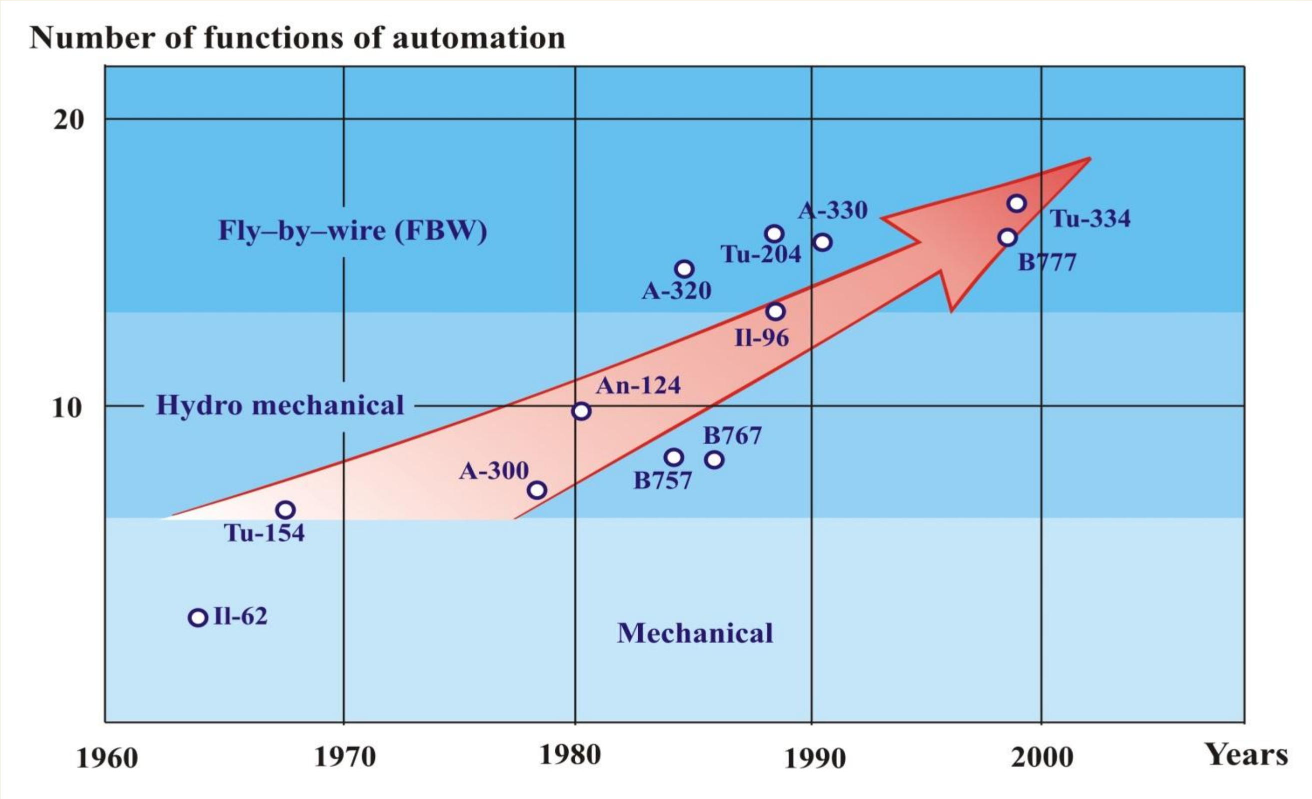
··· PIOR 4÷3 — PIOR 2÷3

c.3. Distribution of control signals between different control surfaces



c.4. Aircraft automation

General tendency



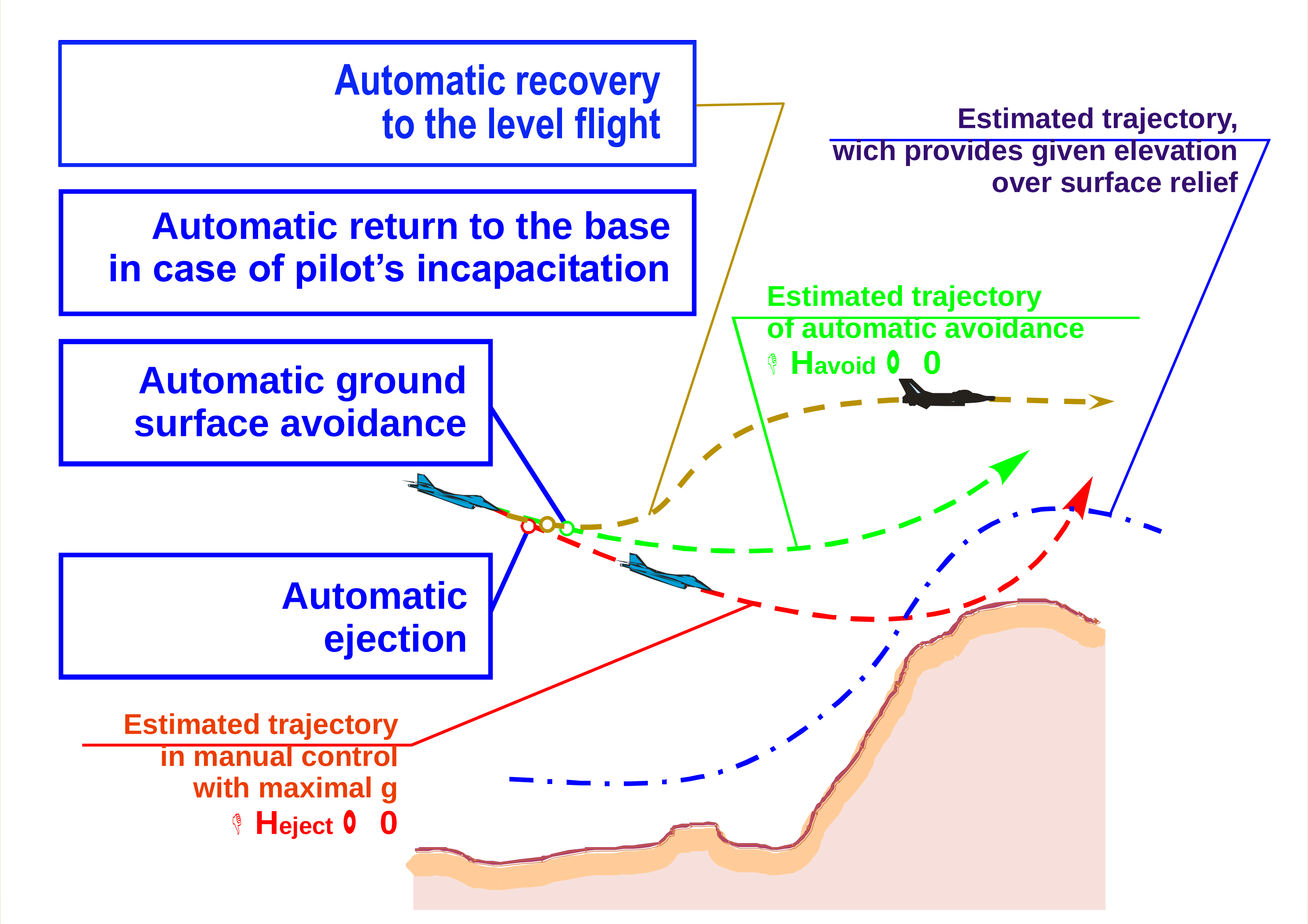
Examples:

– collision avoidance systems;

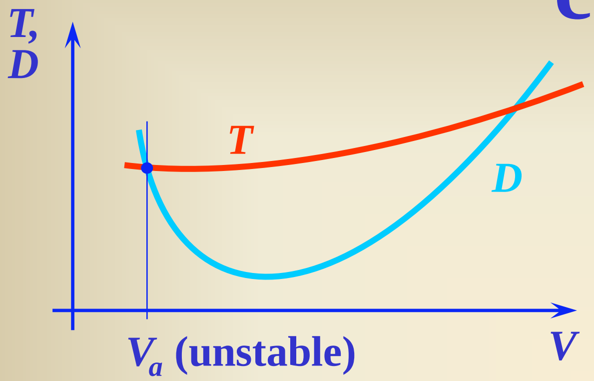
– autothrottle;

–automatic landing

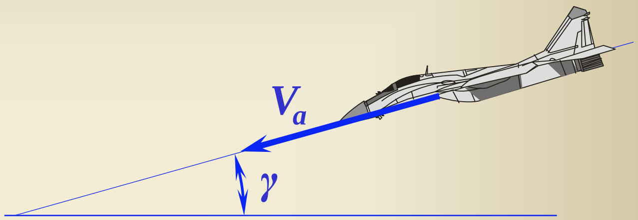
c.4.1. Collision avoidance systems



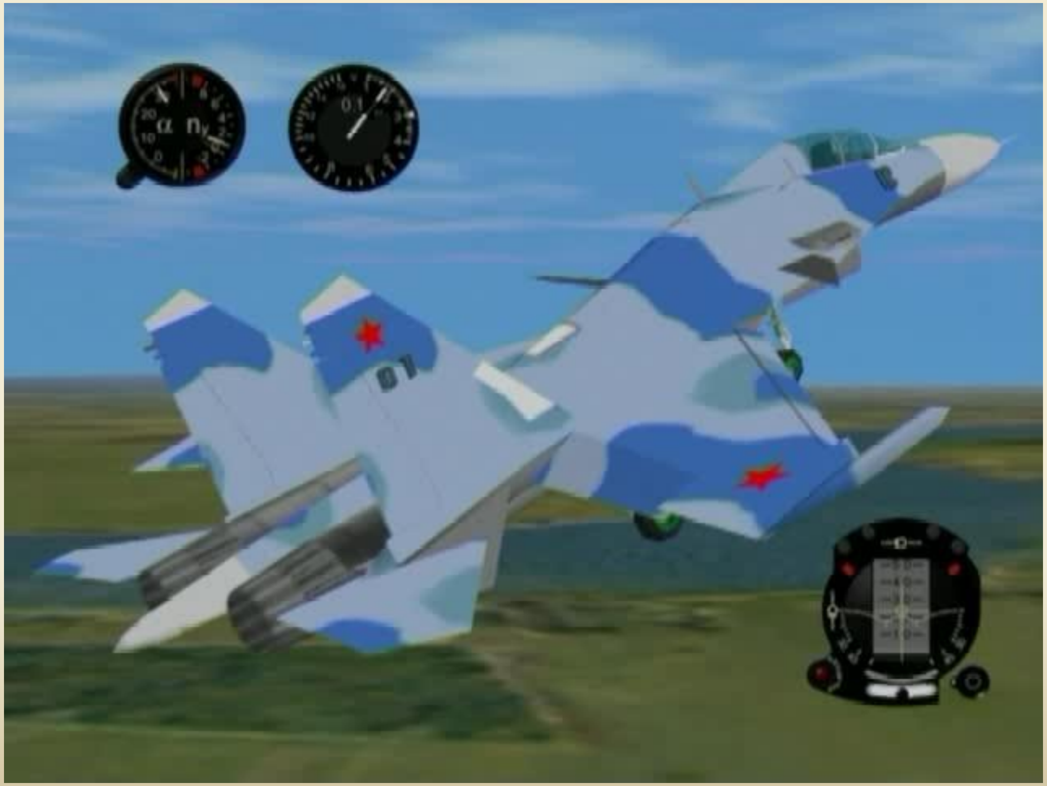
c.4.2. Autothrottle

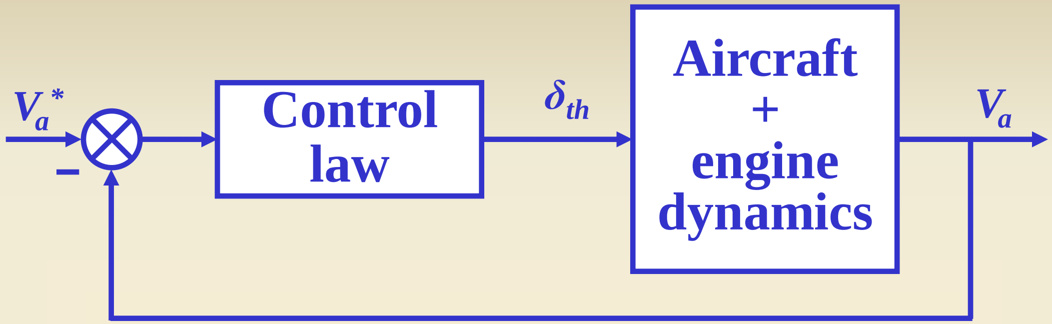


$$\dot{V} = \frac{T - D}{m} - Sin\gamma$$

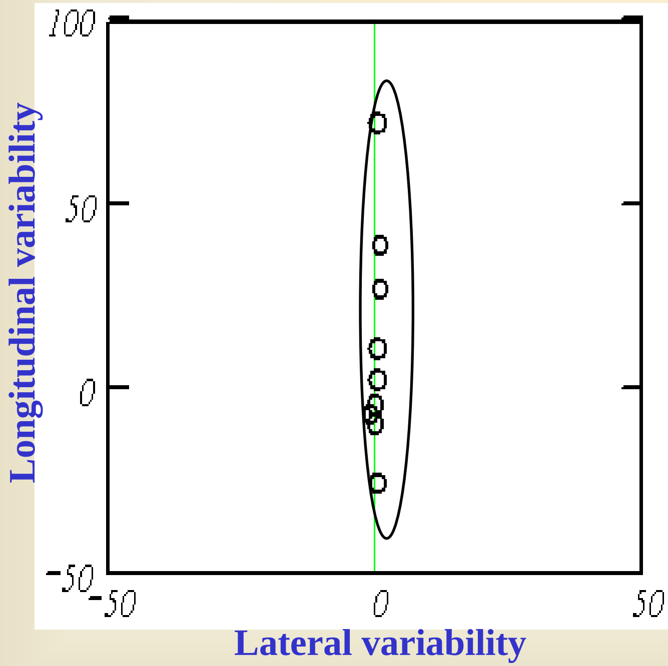


ESTOL

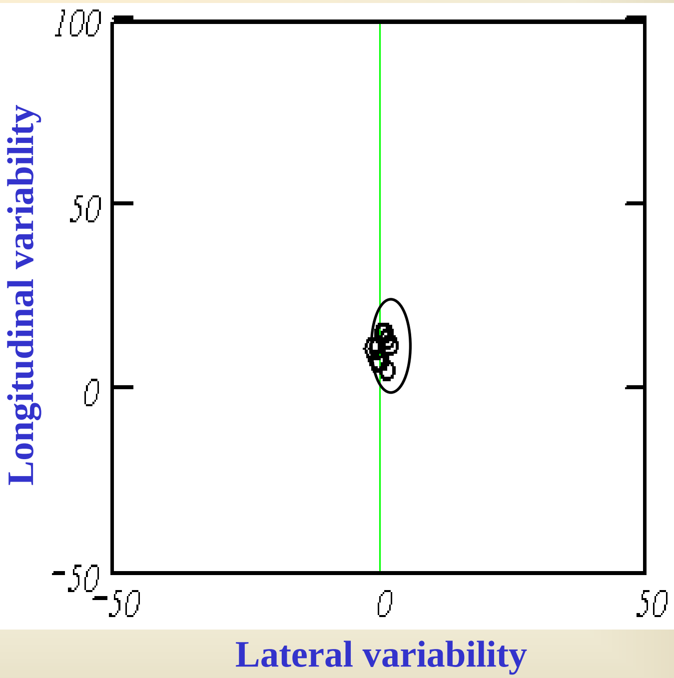




Results of experimental investigations

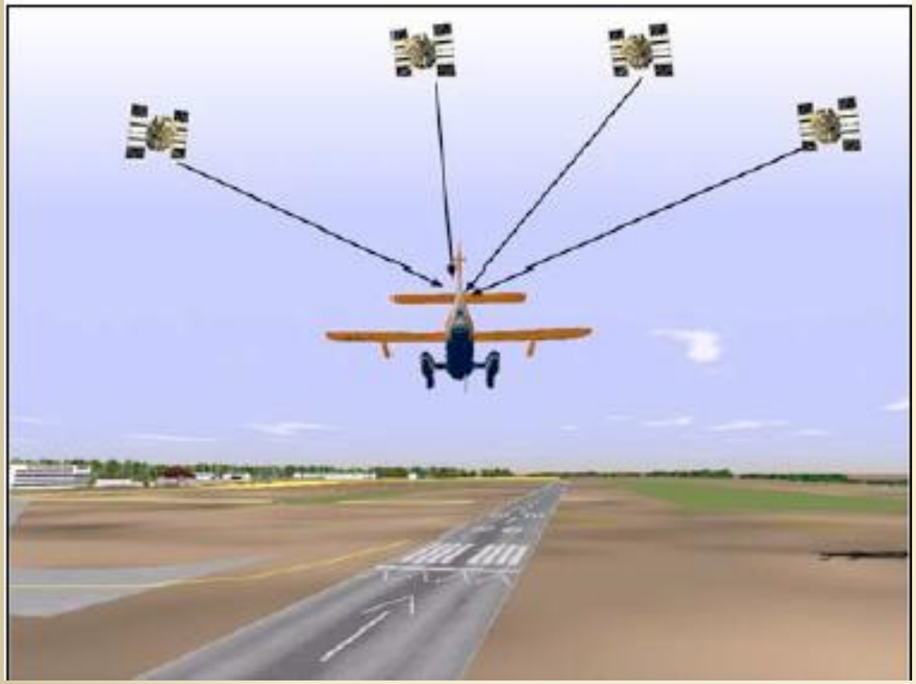


Without autothrottle



With autothrottle

c.4.3. Automatic landing



First worldwide automatic landing based on satellite navigation in Braunschweig (1989)

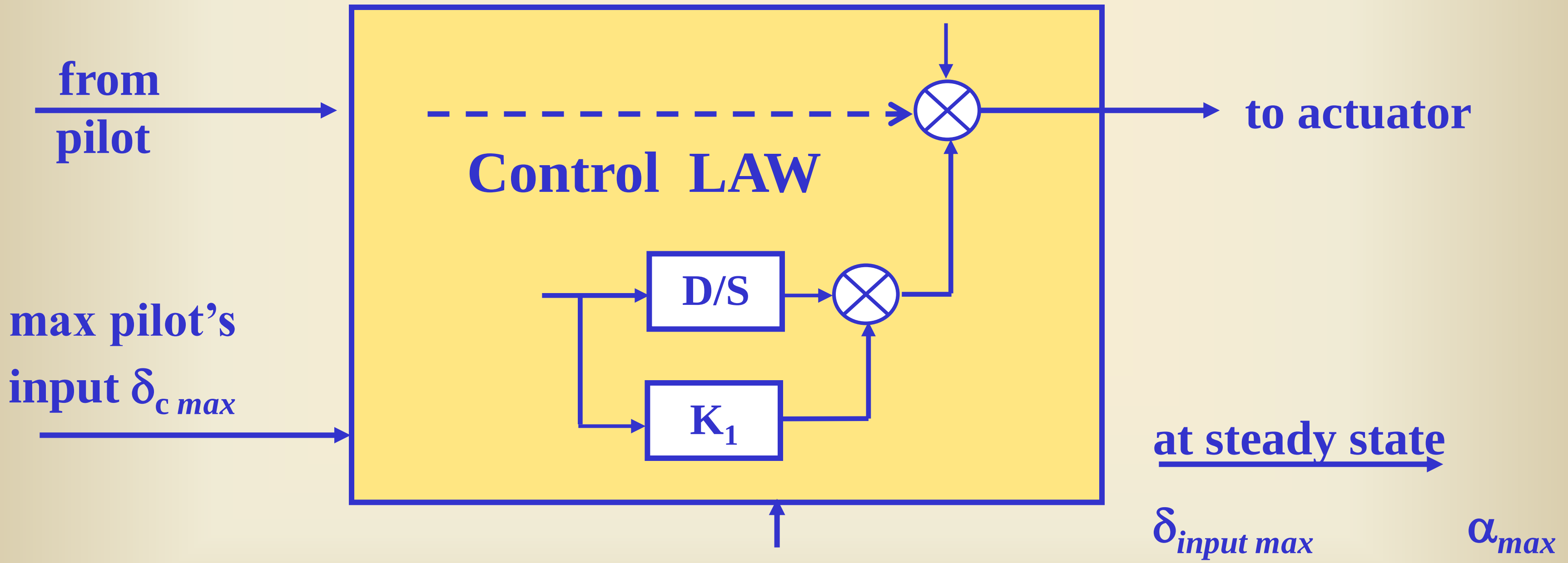
First worldwide automatic landing of aerospace vehicle “Buran” (1989)

c.4.4. Envelope protection systems

– Limiters of pitch and bank angles

– Critical regime warning and barrier system (CRWBS)

Integration of the system with FCS

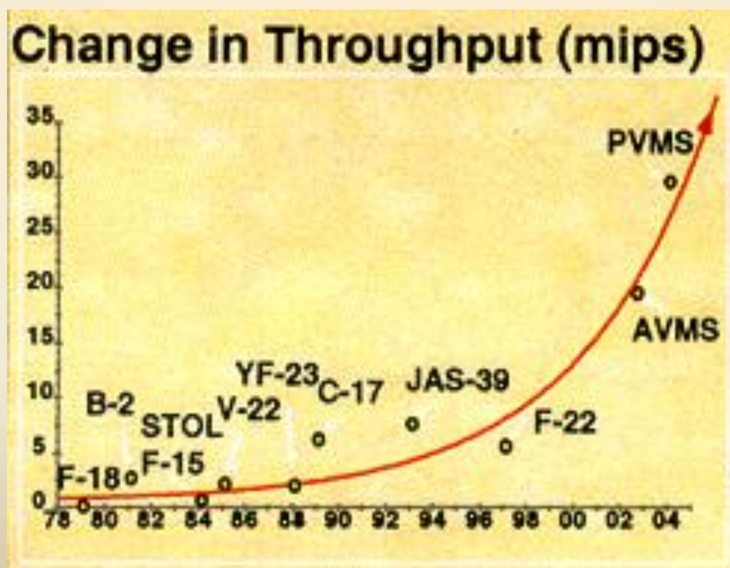


New principle: Change of FCS role and its technology

d) FBW technology

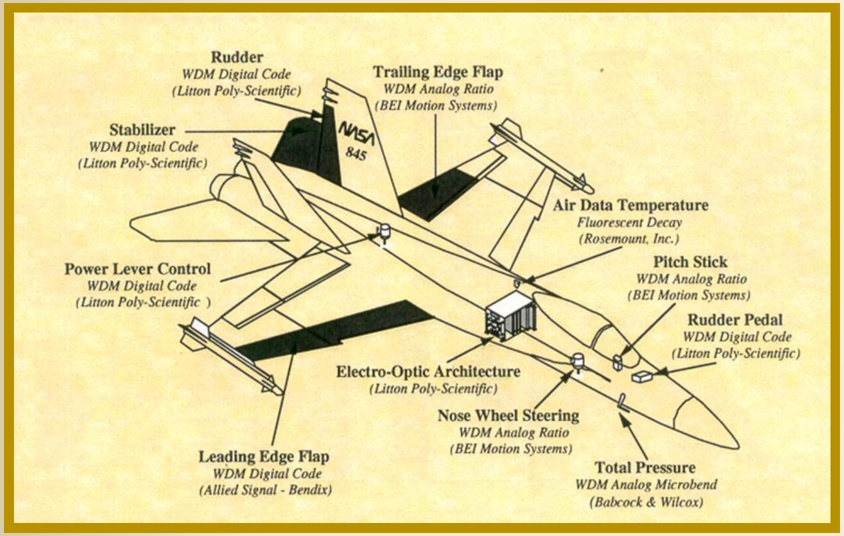
Necessity:

- – increased weight and limitation on throughput (for mechanical linkage);
- – realization of features and potentialities of highly augmented FCS;
- – suppression of nonlinear effects in mechanical linkage

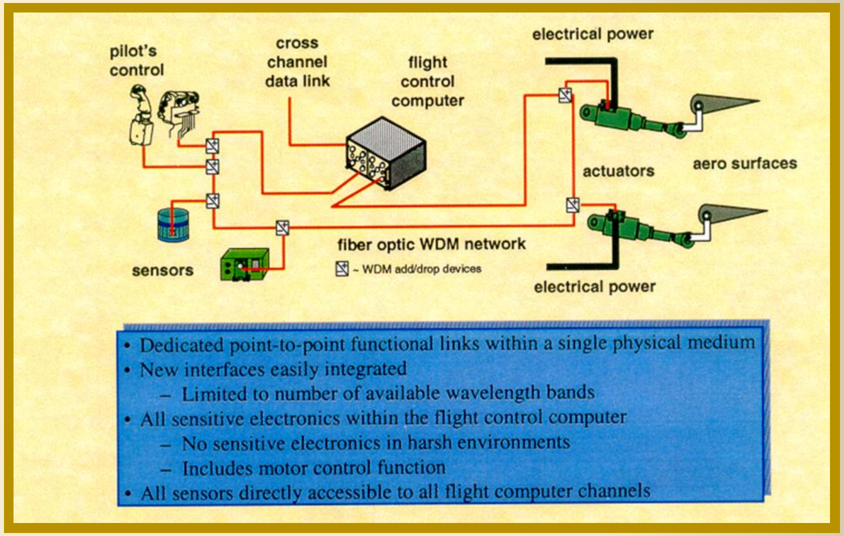


Next Generation — Fly–by–Light technology

First Generation Fly–by–Light



Second Generation Fly–by–Light



The First FBW aircraft –T-4 (100) (Sukhoi company)

First flight – Aug. 1972

Main features:

FBW in all channels

Weight –100 tons

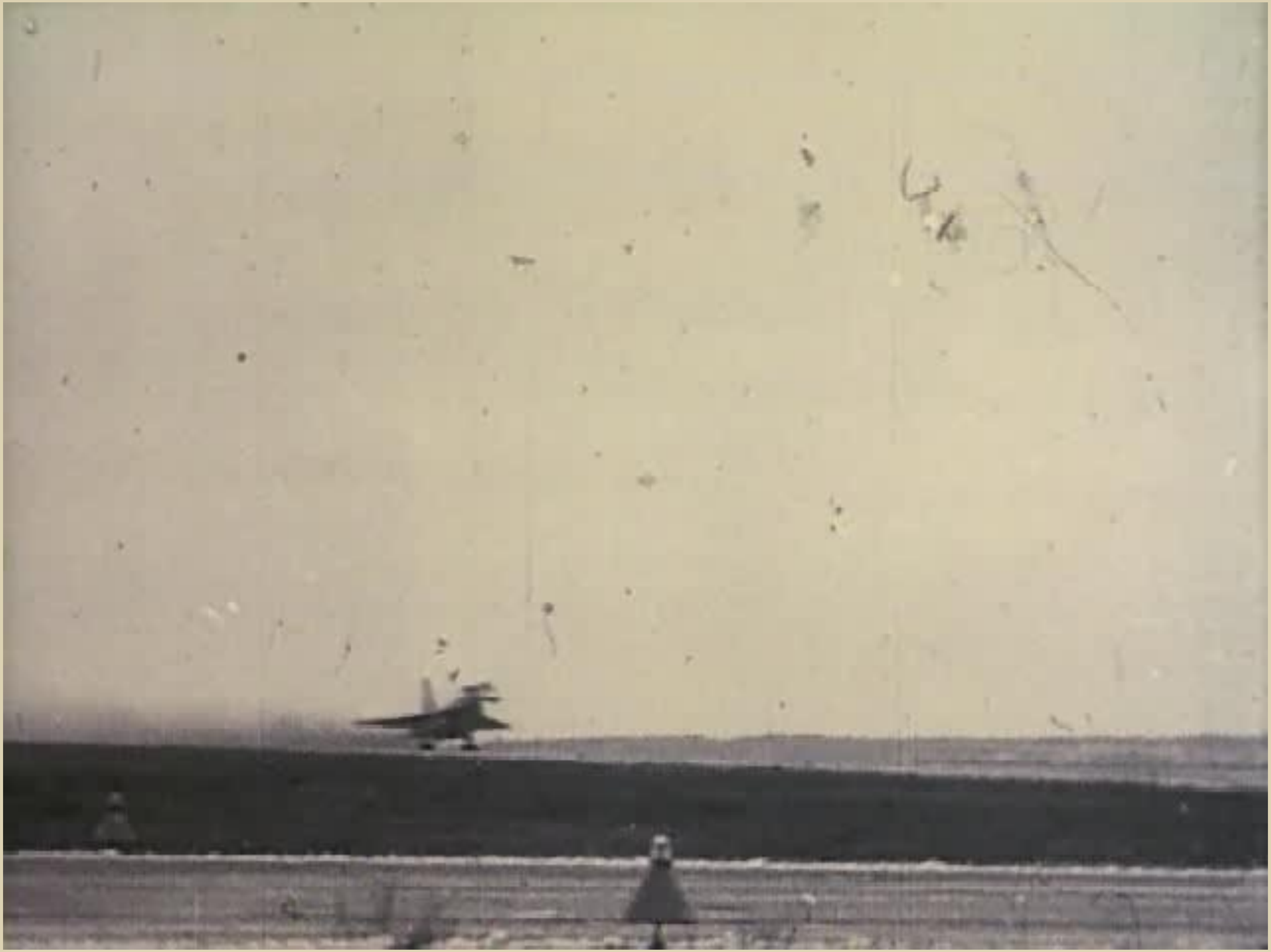
$M_{cruise} = 3$

Additional control surface

Quadruple redundancy

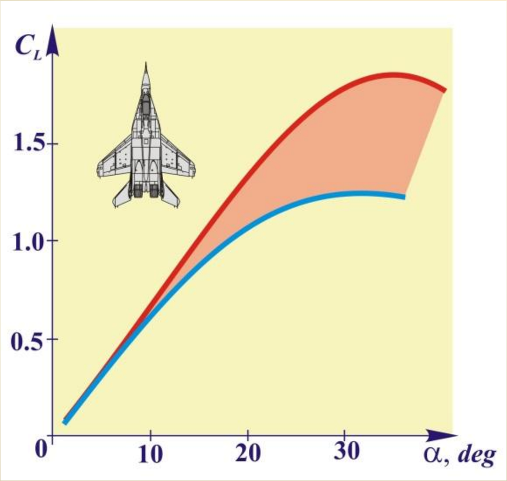
Reduced stability margin – 0% (± 5%)

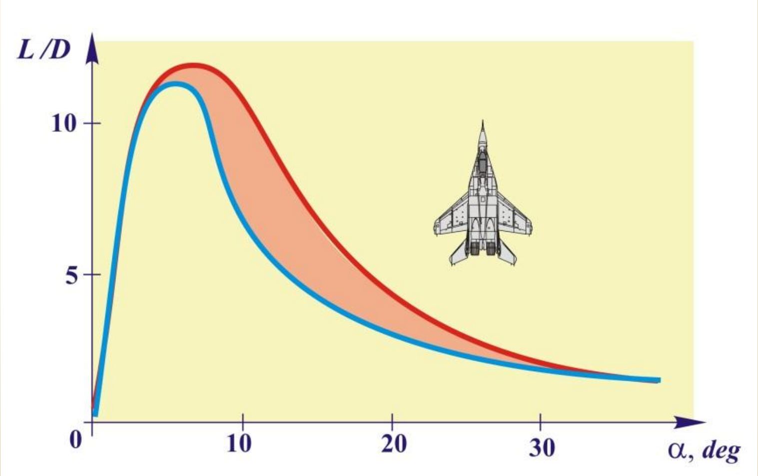
c.g. control system



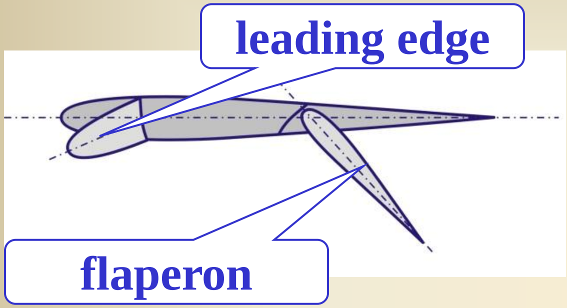
There were developed 20 FBW aircraft in Russia

New principle: Optimization of wing aerodynamics
Innovations:
1. Wing with extension

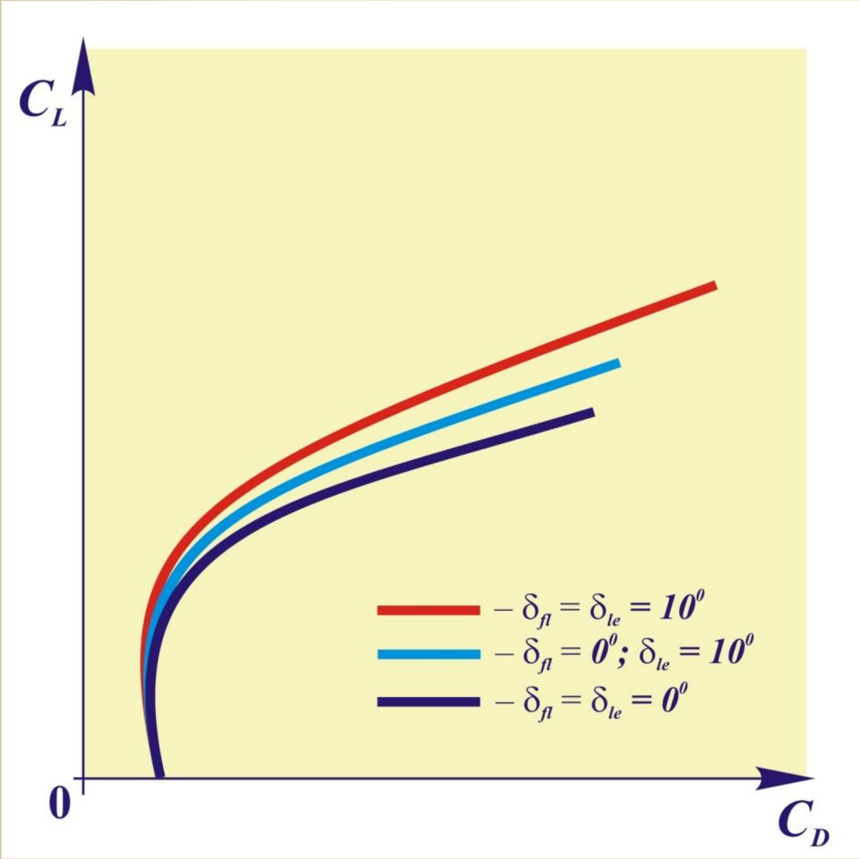




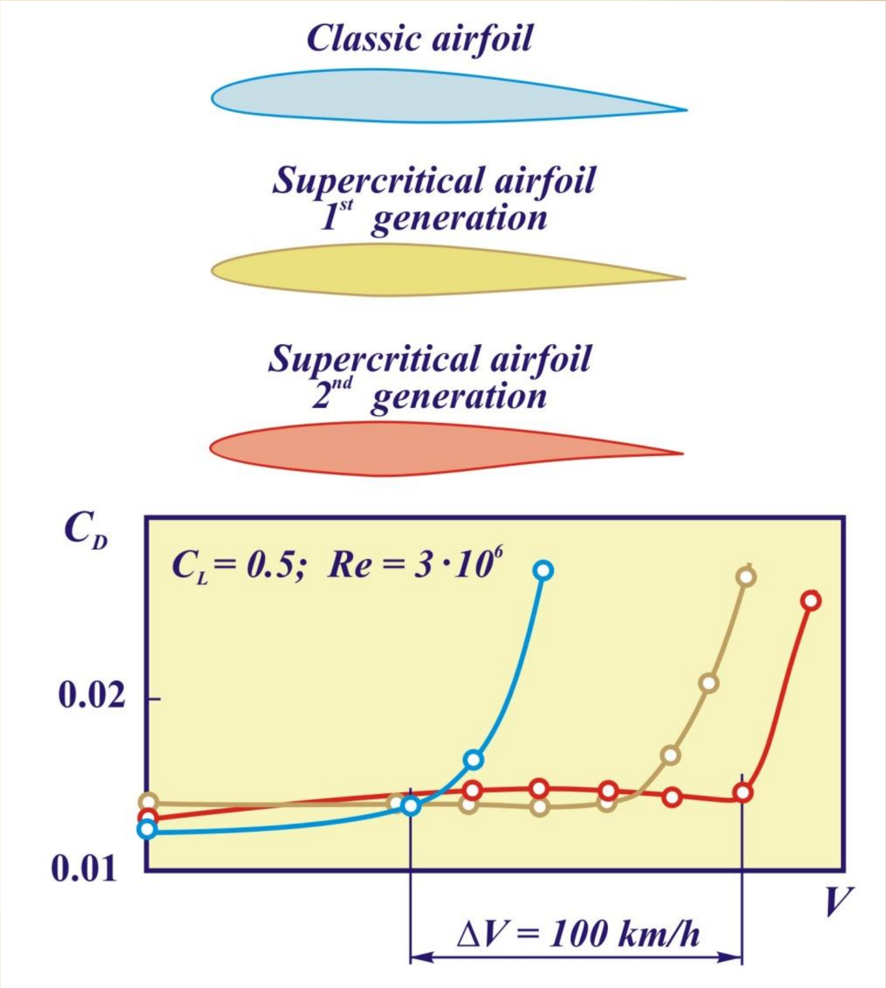
2. Adaptive wing



Deflections of flaperon (δ_{fl}) and leading edge (δ_{le}) are a function of α



3. Supercritical wing



New principle: Super maneuverable flight with unlimited angles of attack

Combination of innovations:

– specific aerodynamics;

– decreased stability;

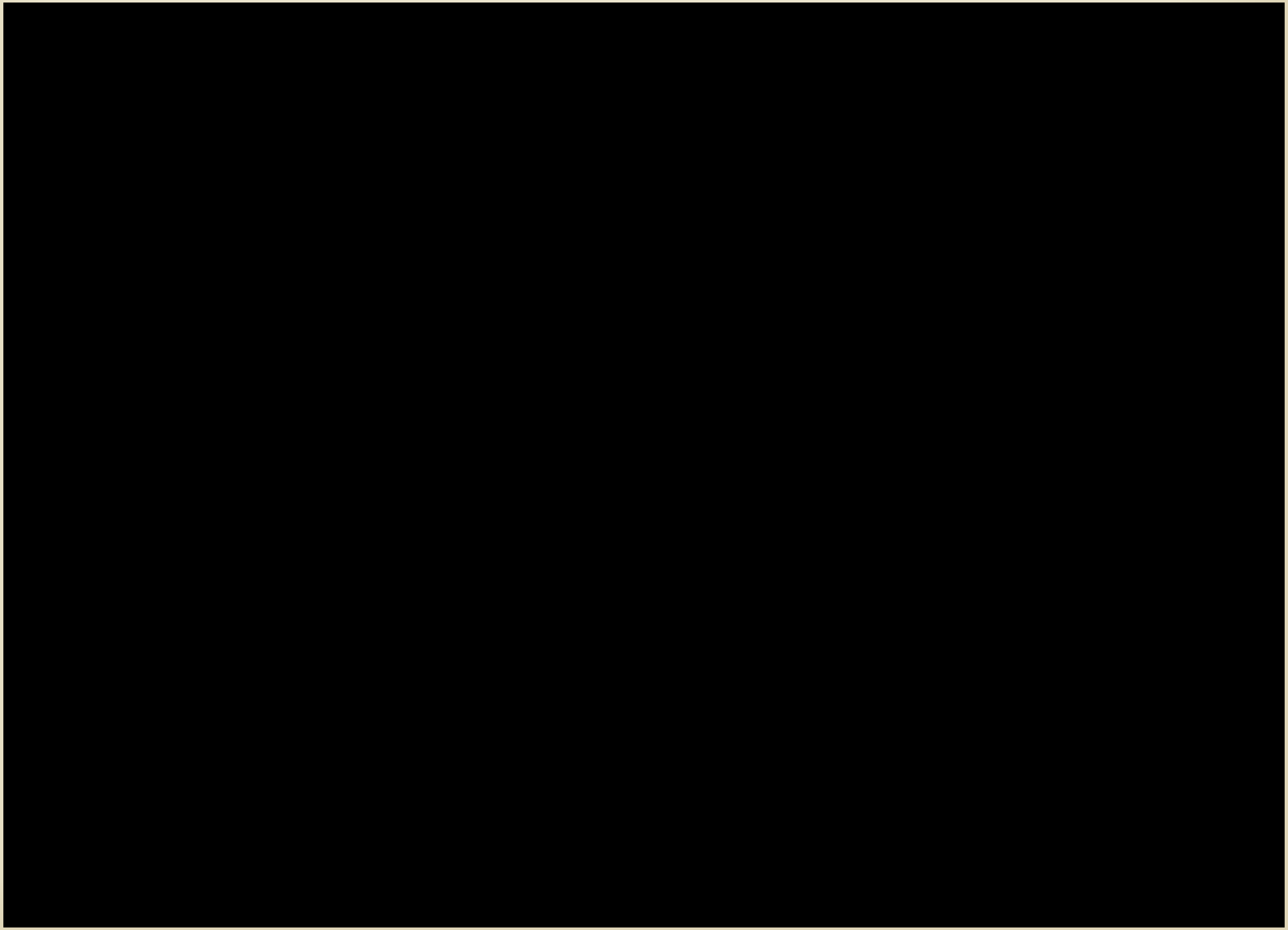
– HA FCS;

–margin of pitch moment for dive;

– thrust vectoring control

Thrust–vectoring control

New maneuvers of super maneuverable aircraft

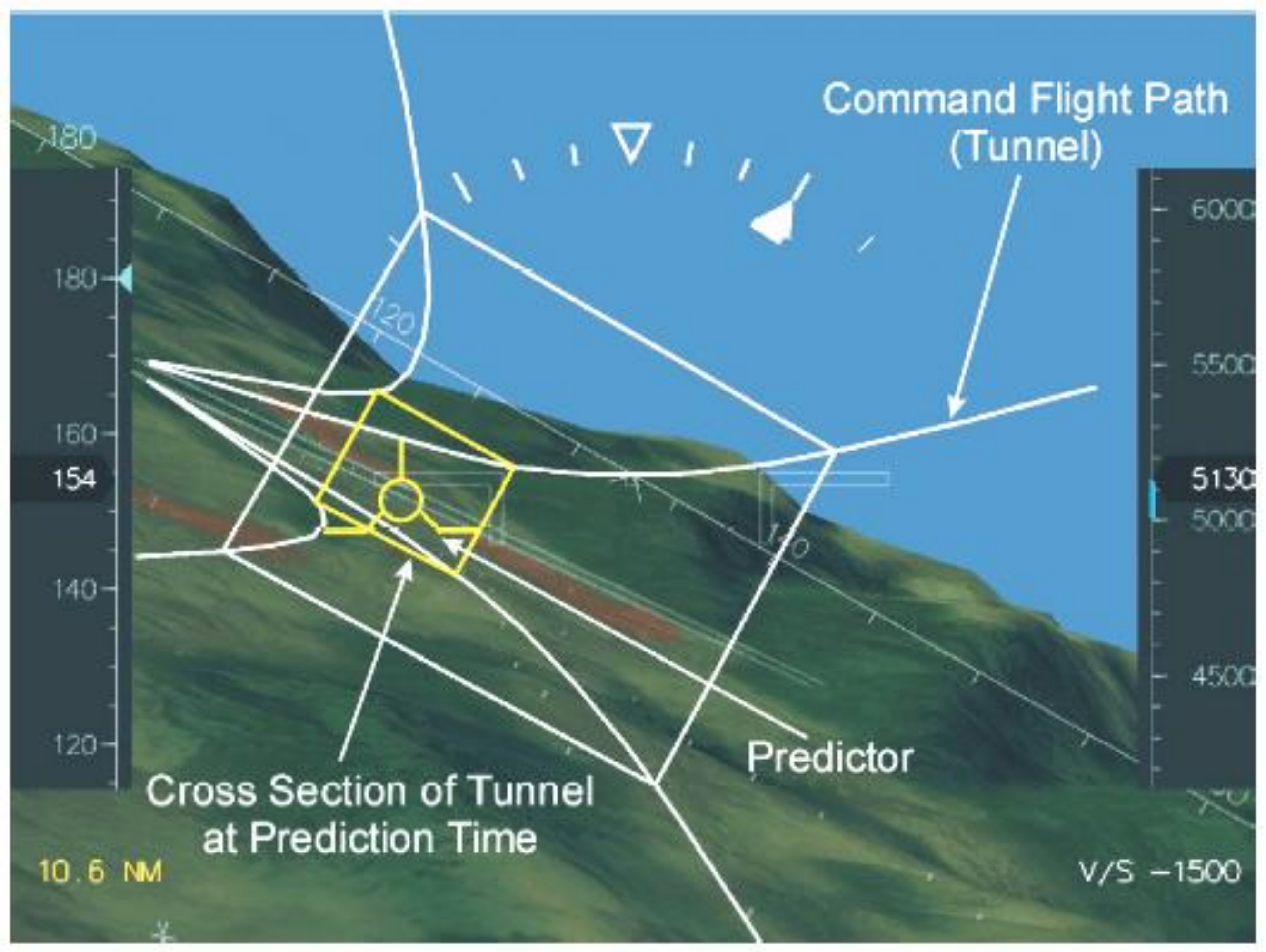


New principle: Interface friendliness

Past

Current

Tomorrow display



New principle: Unmanned air vehicle (UAV)

UAV Vision

- **Enable Autonomous Unmanned Operation For Any Mission**
- **Freely Share The Sky With Manned Aircraft**
- **Attack in Mass to Overwhelm Opponents**

Vision Assumes UAVs As Reliable And Safe As Manned Aircraft
This Is Not The Case Today

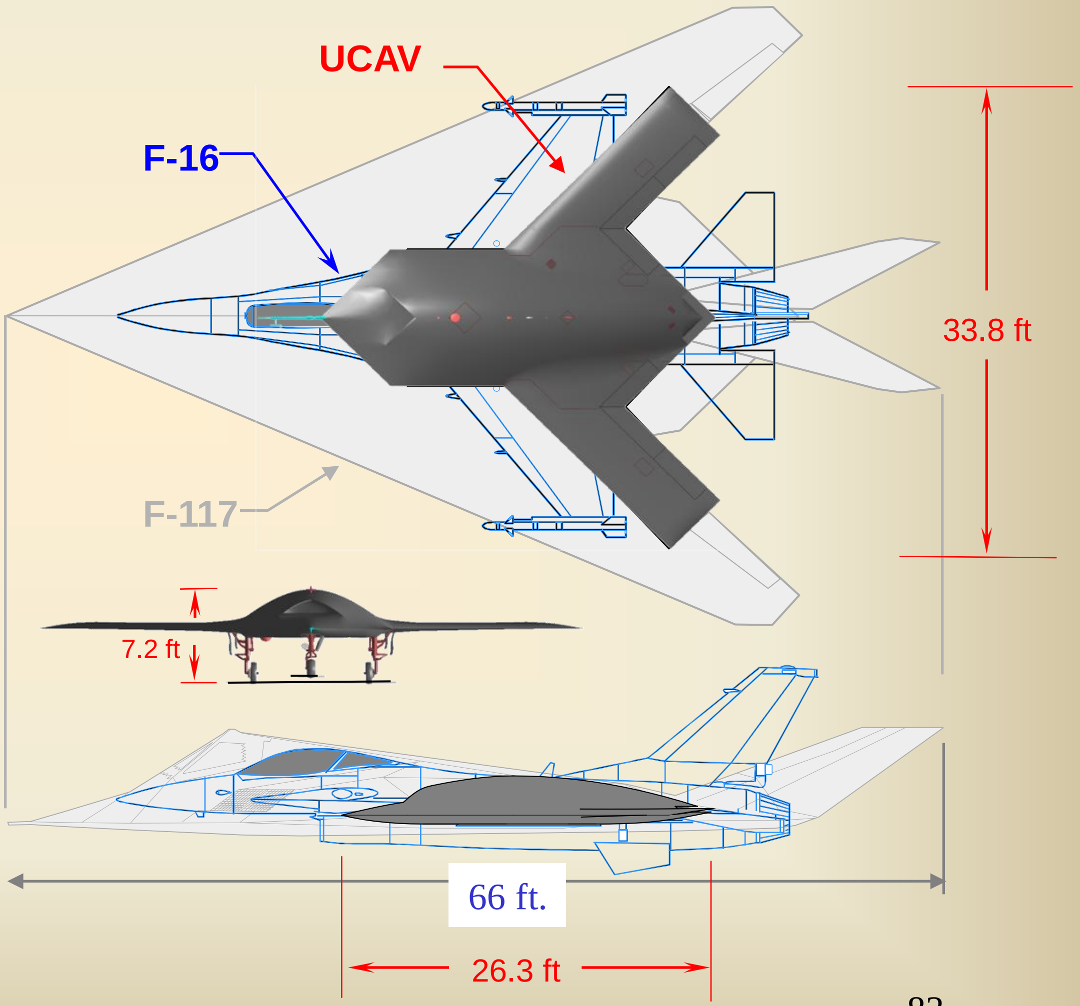
The Autonomy Continuum

Remote Pilot | ***Remote Operator*** | ***Remote Supervisor*** | ***Remote Customer***

None → ***Total***

Increasing Autonomy

Air Vehicle Attributes

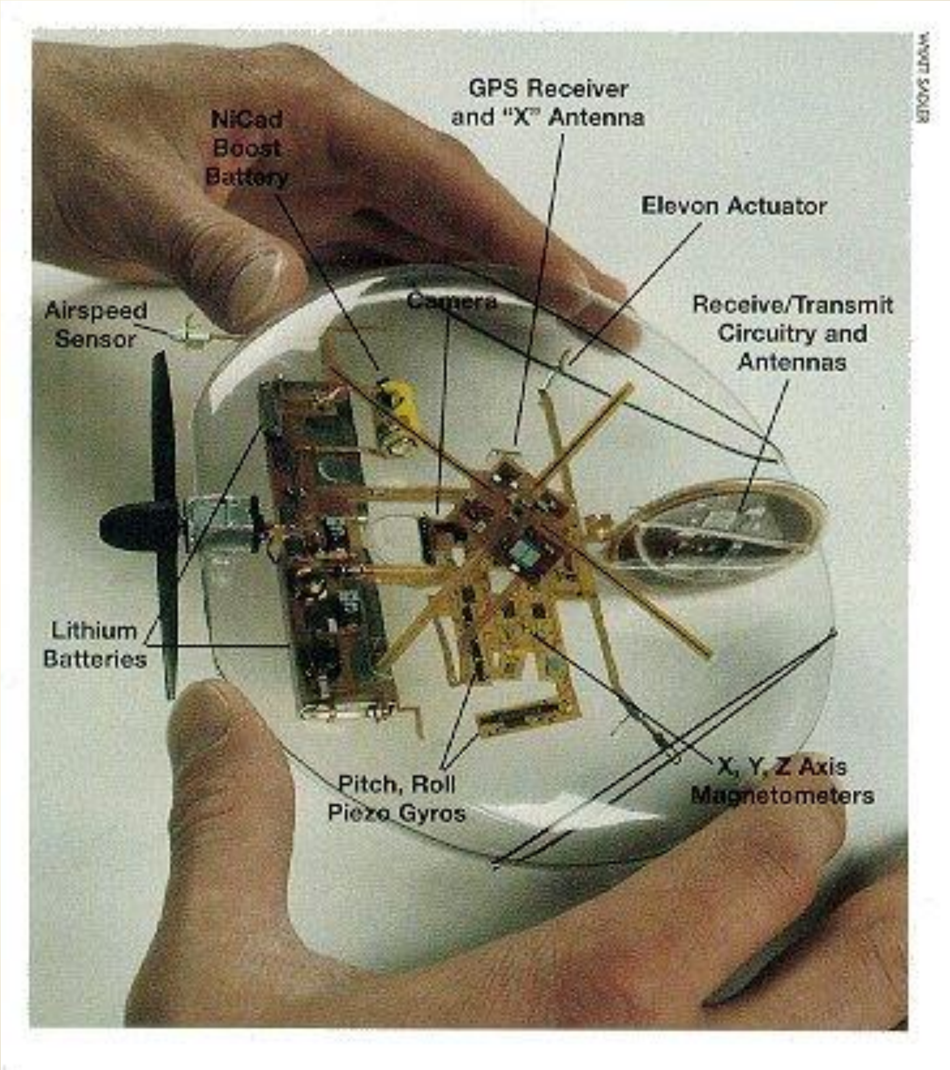


AIR VEHICLE

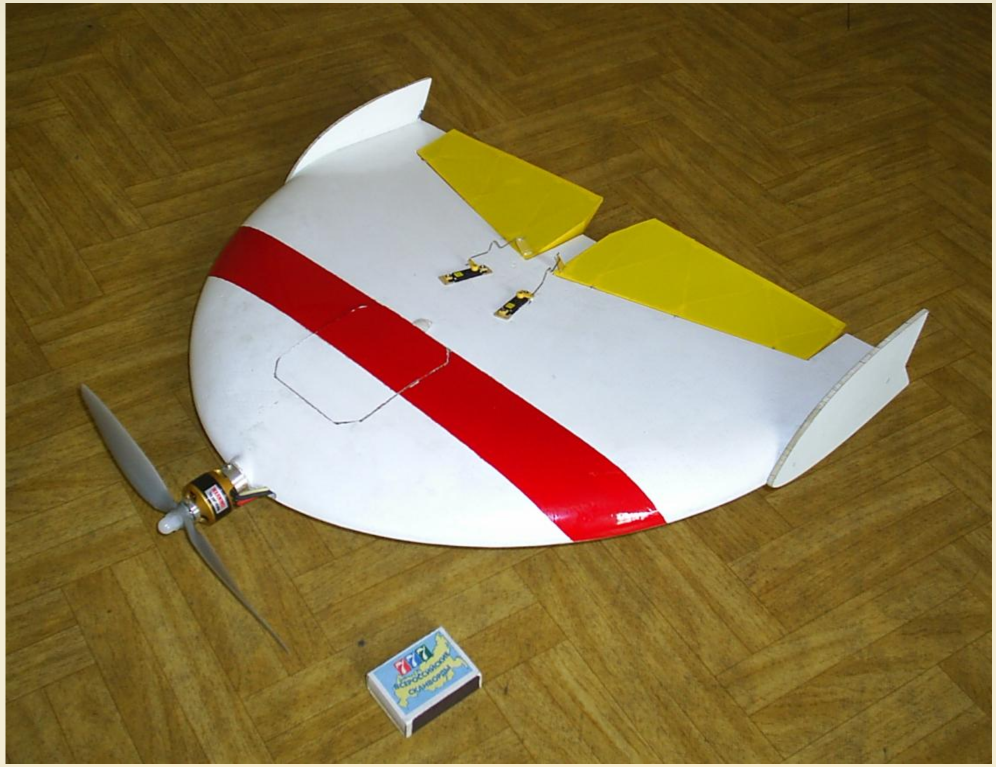
- ~15,000 / 7,500 lb Gross/Empty Weight
- High Subsonic Med/High altitude
- 500-1000 nmi Mission Radius
- 1000-3000 lb Weapons Payload
- Wide range of Current & Advanced Weapons
- All Electric
- Affordable Stealth to the Next Level

Developed Unmanned Combat Vehicle

Miniature Unmanned Aerial Vehicle



Micro aerial vehicle (MAV)



Взлетная масса, кг………...…………………………………..…0,3
Максимальная скорость, км/ч……………………..………….70
Продолжительность полета, ч..………………………… ..до 1,0
Масса полезной нагрузки, кг………………………………..0.03
Двигатель злектро…………………………………………….60 Вт
Назначение………наблюдение за удаленными объектами группами людей, строениями, лесными массивами,

New principle: International cooperation in aero / astro area:

- design and manufacturing;
- research;
- teaching

Example of innovation projects:

JSF X-35

USA, GB, Holland

RRJ

Russia, France, Germany, USA

A-380

France, Germany, GB, Italy, Spain

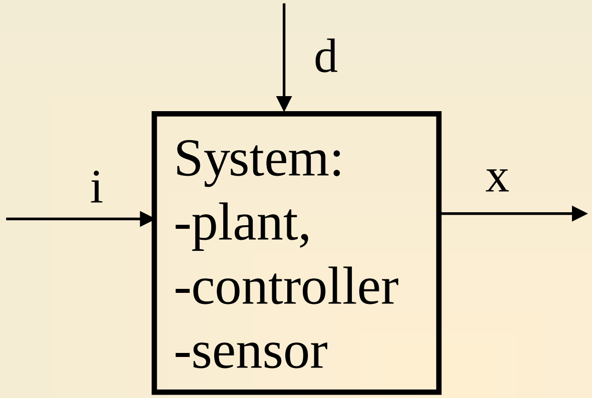
The general principle of any system

The elements of any system:

-plant

-controller

-sensor



General requirements to any system:

-Agreement between output and input signals x≈i

-Low sensitivity to disturbance d(t) $\frac{dx}{dd} \Rightarrow 0$

-Stability (x = $x_{initial}$, when i(t) will return to Zero)

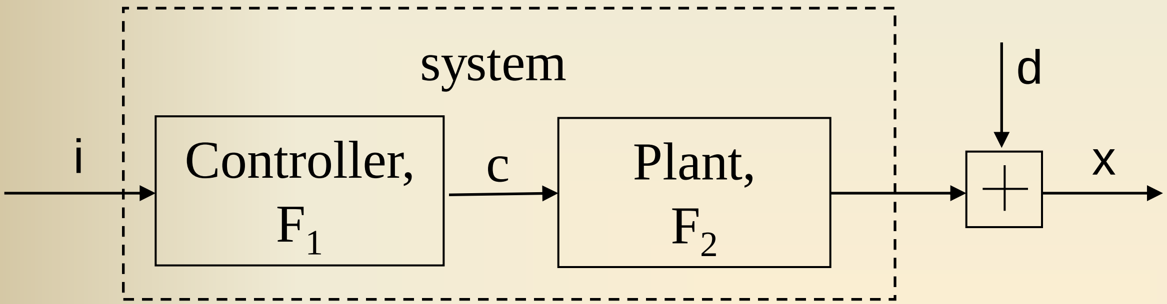
-Suppression of the inaccurate knowledge of the plant dynamics

Plant – aircraft, automobile, ship …

Controller (autopilot, pilot,…) applies the control action (energy) to the plant according to the rules in order to make specified system responses conform as closely as possible to some standard or criterion

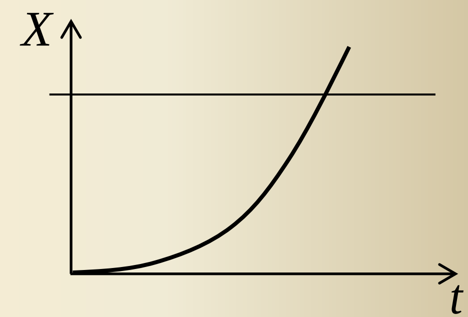
Two types of the system

Open-loop system



Example:

$F_2 = \int \Rightarrow F_1$ has to be equal to $\frac{d}{dt}$



a) d=0; i≠0; $F_1 = \frac{1}{F_2};$ ***to get x=i***

b) i=0; d≠0; x=d

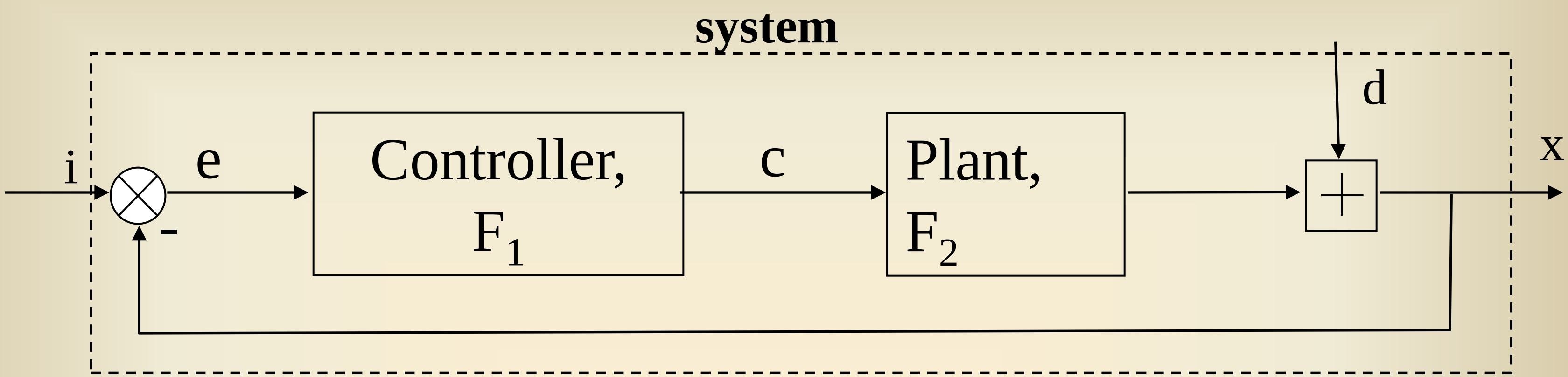
c) if F_1, F_2 unstable $\frac{1}{s-a} \rightarrow$ The aircraft id divergent

d) $F_2 = F^* + \Delta F$ $X = iF_1F_2 + \Delta F F_1 i$

Conclusion:

- **controller law is too complicated;**
- **open-loop system does not suppress a disturbance**
- **the instability can not be suppressed**
- **Impossibility to suppress the inaccurate knowledge of the plant dynamics**

Closed loop system



a. $d = 0 \quad \frac{x}{i} = \frac{F_1F_2}{F_1F_2+1}\Big|_{F_1F_2>>1} \cong 1$

$b.$ $i = 0 \quad d = 0 \quad \frac{x}{d} = \frac{1}{1+F_1F_2}\Big|_{F_1F_2>>1} \cong 0$

$Example: \; if \quad F_2 \doteq \int \; then \; F_1 = K, \; (K >> 1)$

c. Provision of stability

$$F_2 = \frac{1}{s-a}$$

$$F_2 = a$$

$$\frac{X}{i} = \frac{a}{s+(a-0.1)} i(t)$$

$a > 0.1$ system is stable

$$F_2 = F^*(s) + \Delta F(s)$$

$$X = \frac{F_1F_2 + \Gamma(s)}{1+F_2[F_1+\Gamma(s)]} i$$

$$for \; F_1F_2 >> 1, \quad y \cong i$$

In closed - loop system:
- controller law is simpler considerably;
- the disturbance might be suppressed;
- provision of stability of the system for unstable plant;
- suppression of the inaccurate knowledge of the plant dynamics

Different types of controller

$$F_1 = \frac{v(s)}{d(s)} \qquad F_2 = \frac{b(s)}{a(s)} \qquad y(s) = \frac{b(s)v(s)}{a(s)d(s)+b(s)v(s)} i(t)$$

1. $(v(s) = K; \;\; d(s) = 1) \Rightarrow u(t) = K_c(i - y)$ - proportional type

$$X\Big|_{t\to\infty} = \lim_{s\to\infty} \frac{b(s)\mathrm{K}}{a(s)+b(s)K}\Bigg|_{K>>1} \approx 1$$

2. $d(s) - 1 \qquad v(s) = K_D s \qquad u(t) = K_D\, \mathrm{s}[\dot{i}(t) - \dot{\mathrm{y}}(\mathrm{t})]$ PD - controller

$$X\Big|_{t\to\infty} = \lim_{s\to 0} \frac{Ksb(s)}{a(s)+Ksb(s)} = 0$$

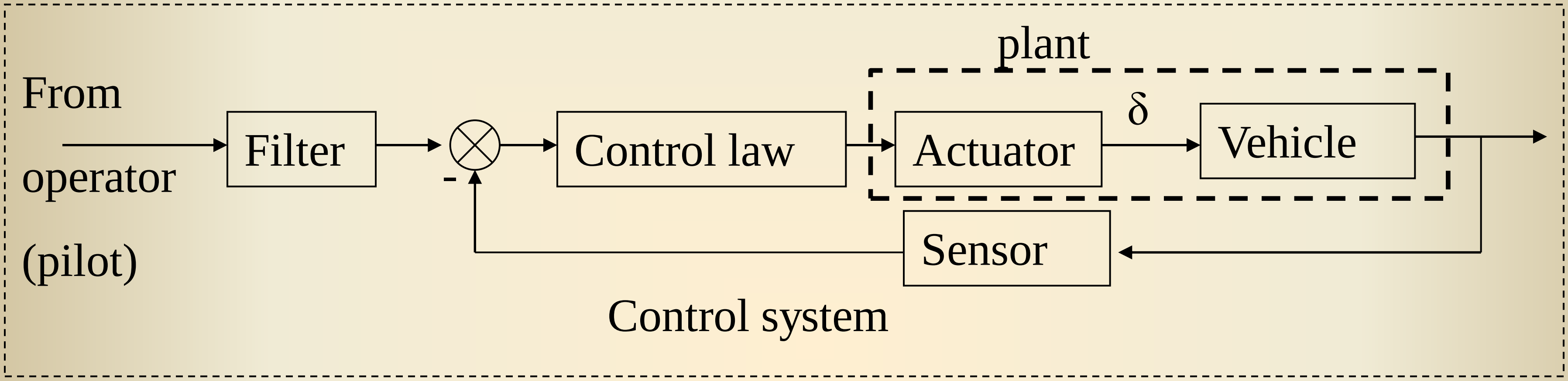
3. $d(s) - s \qquad v(s) = K$ - Integrator control

$$X\Big|_{t\to\infty} = \lim_{s\to 0} \frac{b(s)\mathrm{K}}{\underset{\substack{\downarrow \\ 0}}{a(s)\mathrm{s}}+b(s)K} = 1$$

Task variables:

Controlled element dynamics –

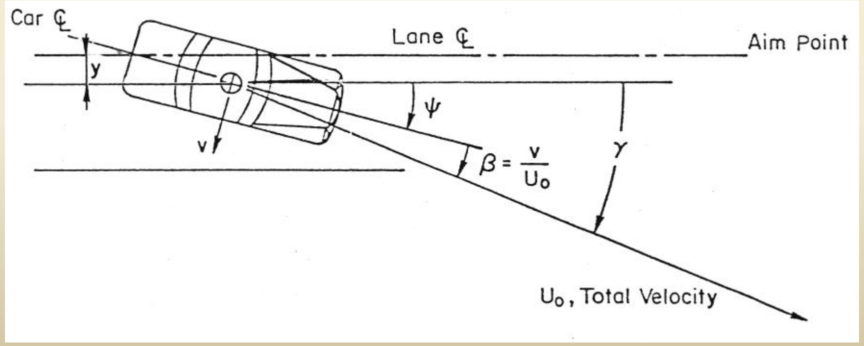
Dynamics of the system: vehicle + control system



VEHICLE:

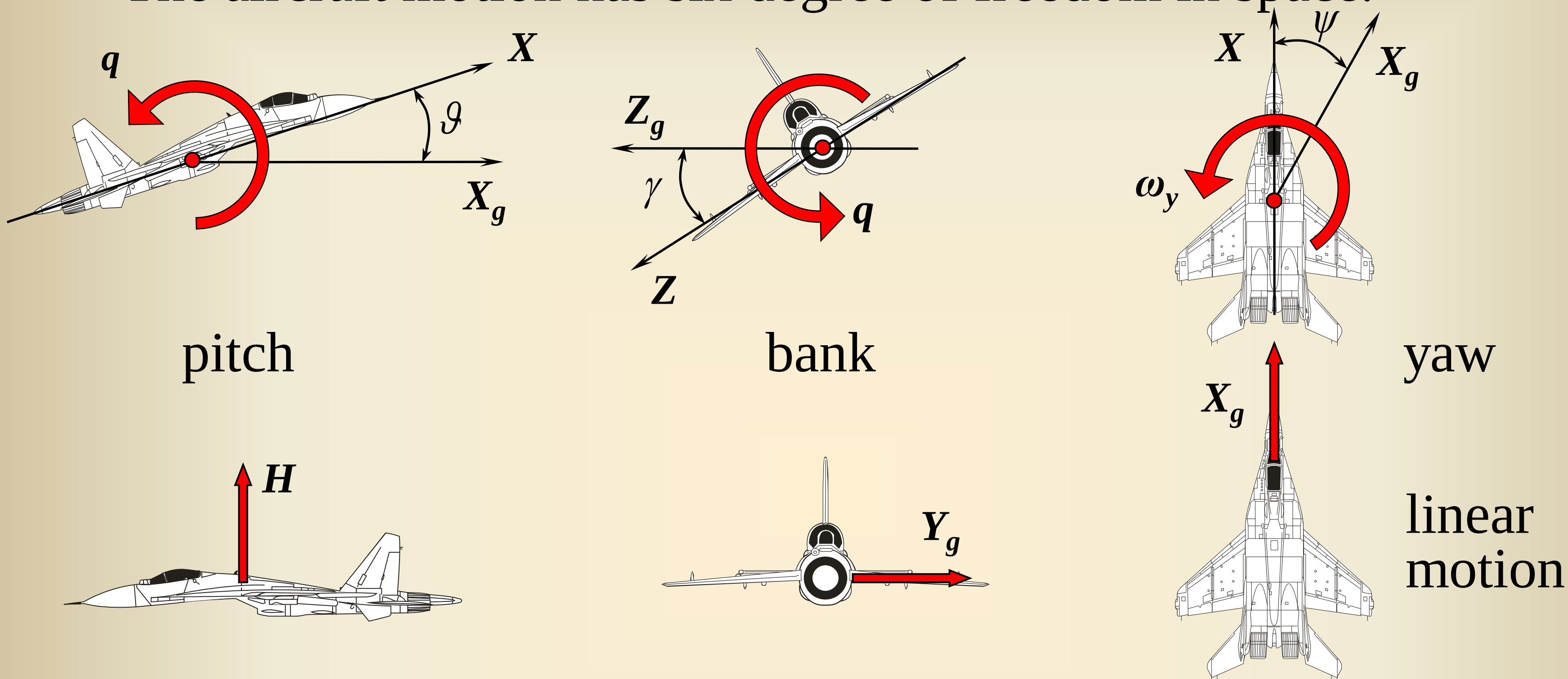
AUTOMOBILE DYNAMICS

3 degree of freedom (ψaw, lateral, longitudinal)



AIRCRAFT DINAMICS

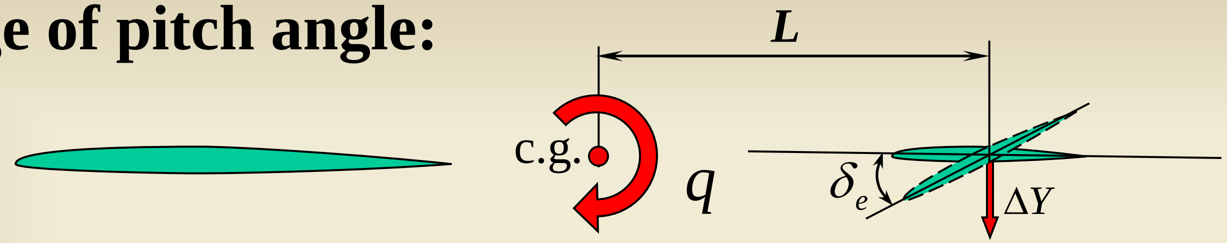
- The aircraft motion has six degree of freedom in space.



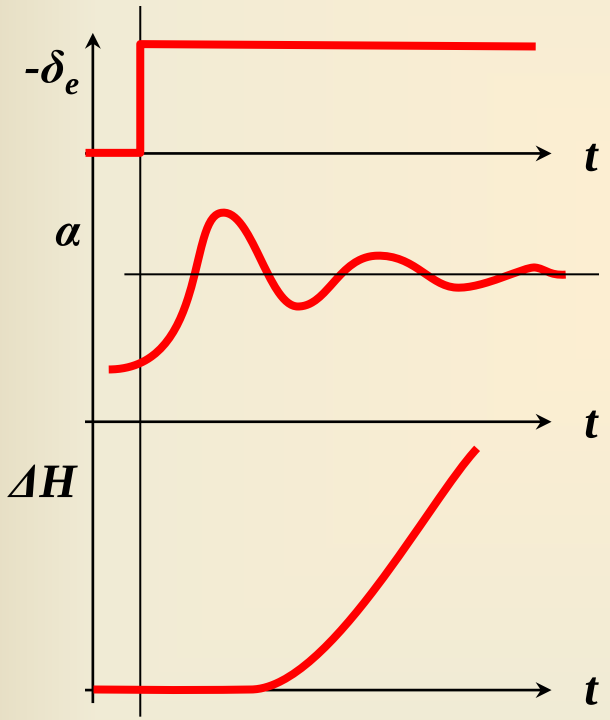
- The rotations of aircraft (***ω***) lead to the change of angles.
- The reasons of the rotations are the applied moments
- Moments are aroused by the deflections of control surfaces (***δ***)
- **The linear displacements** are aroused by the change of angular position.
- **The angular and linear motions are coupled**

Coupling of linear and angular motion

Change of pitch angle:



$$\delta_e \Rightarrow \Delta Y \Rightarrow M_z = \Delta L \cdot L \Rightarrow q \Rightarrow \theta(t) \Rightarrow \alpha(t)$$



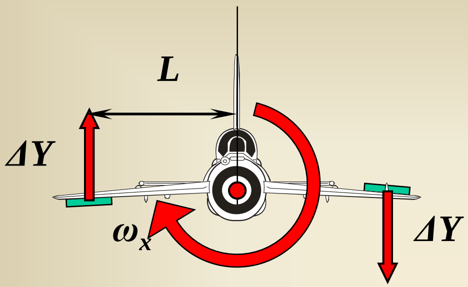
$$\alpha(t) \Rightarrow \Delta Y \Rightarrow (H)$$

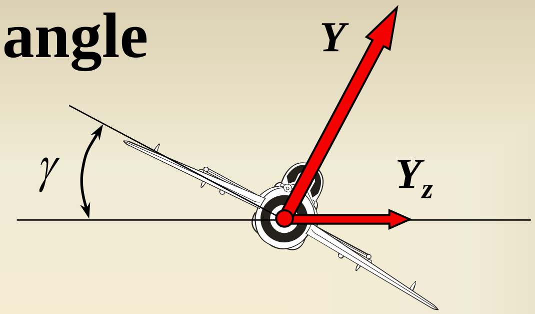
change θ $\Rightarrow$ change of ***H***

(through the change of ***α***)

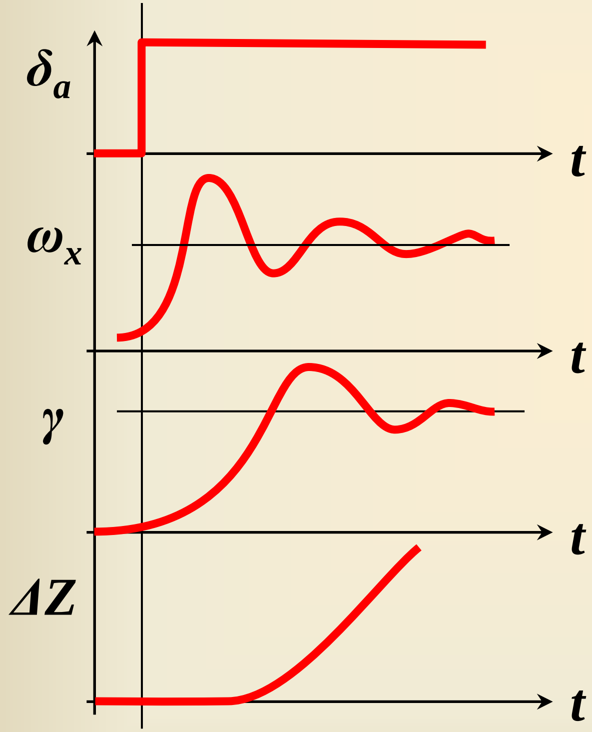
The **elevator** is a control surface for change of **pitch angle and altitude**

Change of bank angle





change of aileron position – moment



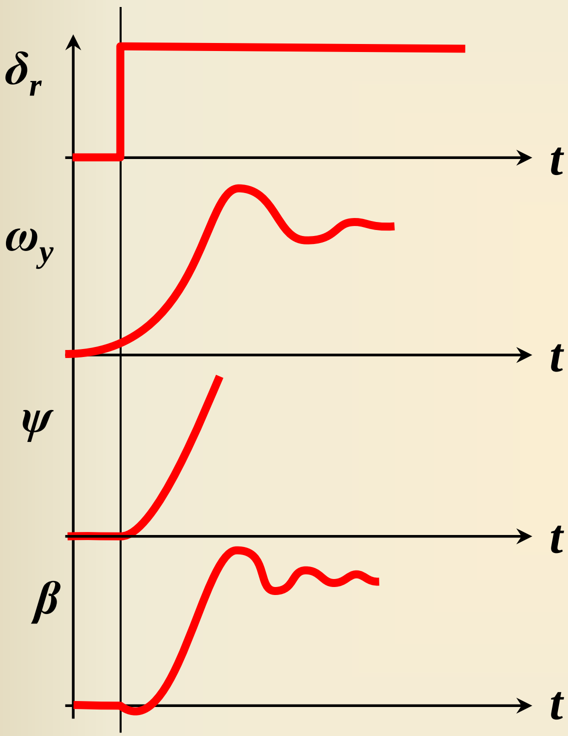
$$M_x = 2L \cdot \Delta Y \Rightarrow \omega_x \Rightarrow \gamma(t)$$

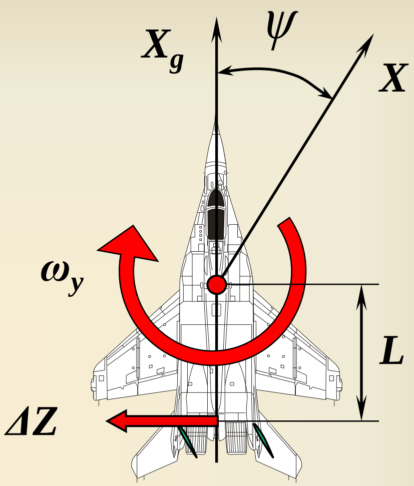
$$\gamma(t) \Rightarrow Y_z = Y \sin\gamma \Rightarrow Z(t)$$

Change of γ ⇒ change of Z
(through the component of Y)

Aileron is a control surface for change of bank angle and lateral position

Change of yaw angle





$$\delta_r \Rightarrow L \cdot \Delta Z \Rightarrow \omega_y \Rightarrow \psi(t)$$

$$\psi(t) \Rightarrow \beta(t) \Rightarrow Z(t)$$

Change of $\boldsymbol{\psi(t)}$ $\Rightarrow$ change of $\boldsymbol{Z}$ (through change of $\boldsymbol{\beta}$) uses rather seldom.
Rudder is a control surface for change yaw angle (and $\boldsymbol{\beta(t)}$)

Velocity control

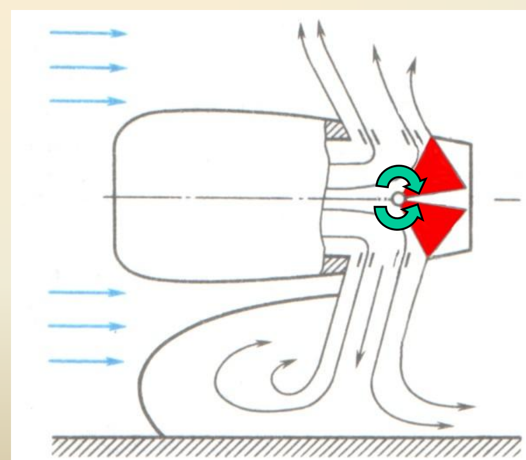
a. Thrust control.

Regulation of fuel ⇒ thrust ↑↓ ⇒ velocity ↑↓.

b. Devices for deceleration.

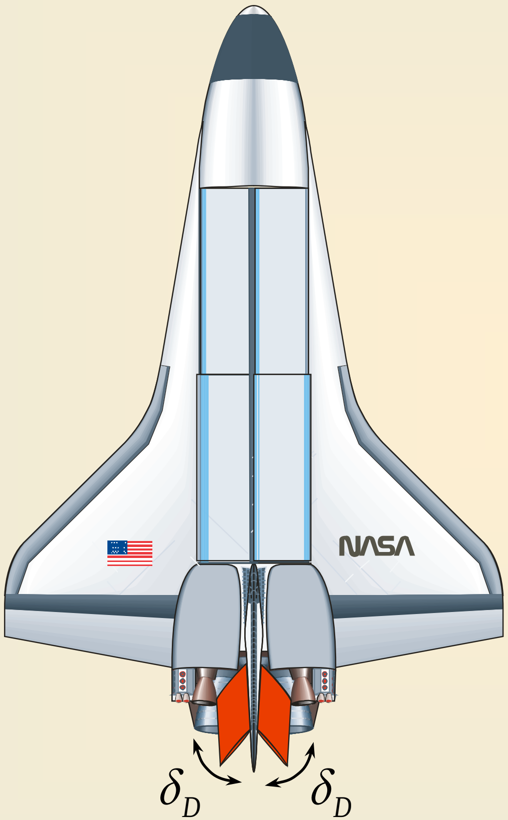
– parachute

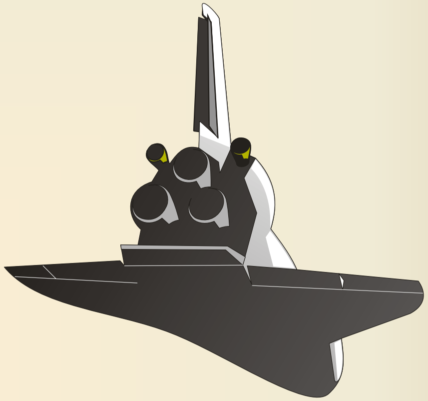
– reverse thrust



Velocity control

c. Aerodynamic deceleration surface.



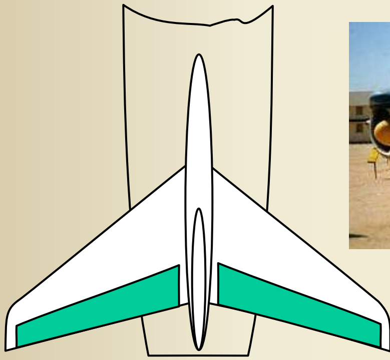


$$\delta_D \updownarrow \Rightarrow C_x(D) \updownarrow \Rightarrow V \uparrow\downarrow$$

Space Shuttle, «Buran».

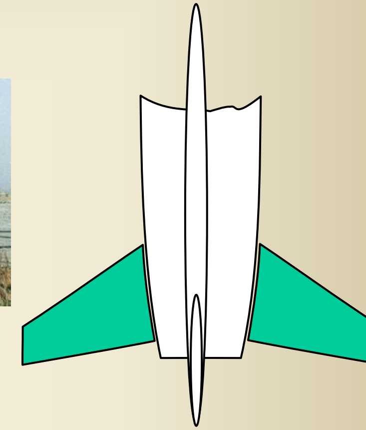
Different control surfaces used for control

In longitudinal channel.



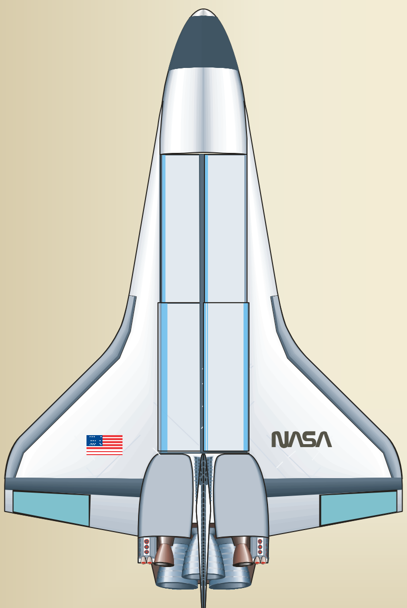
MiG-17

MiG-21



elevator

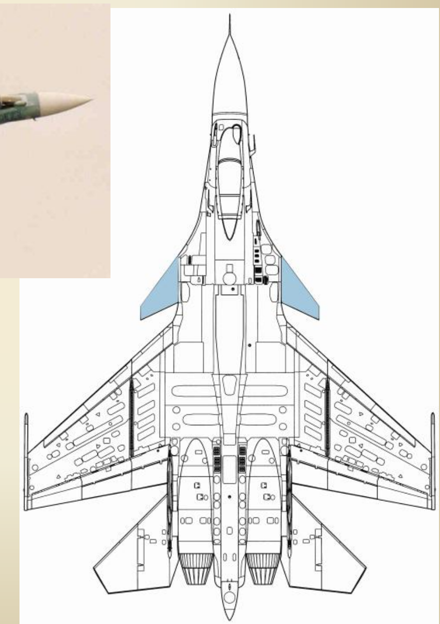
stabilizer



elevons

Concord, Tu-144, «Buran», Space Shuttle

Su-33, Su-35, Eurofight er

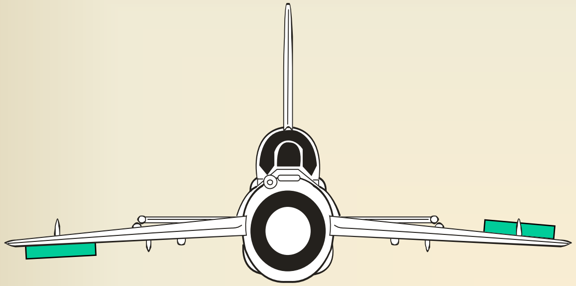


canard

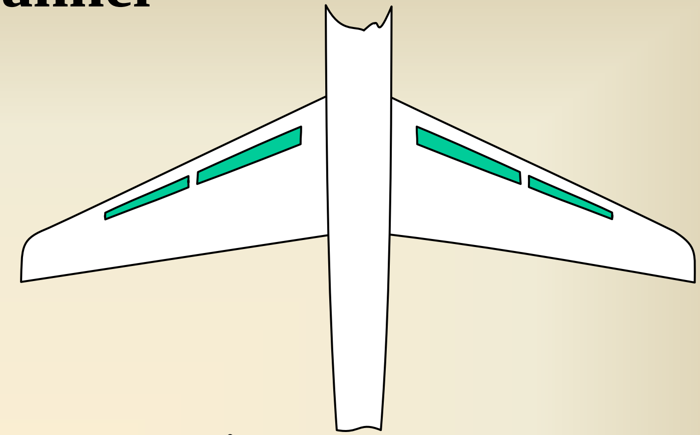
Thrust vectoring control

In lateral channel

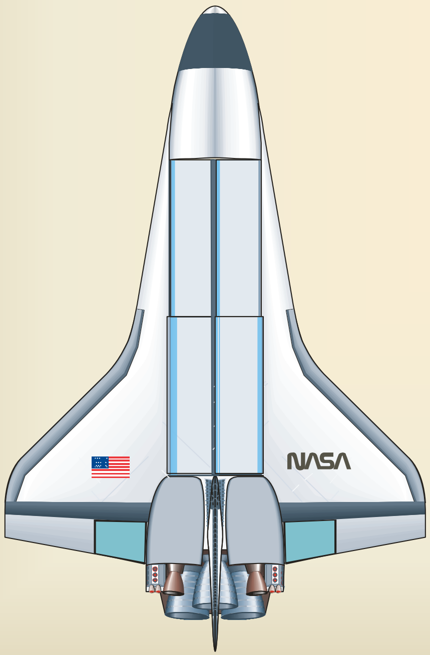
Bank control



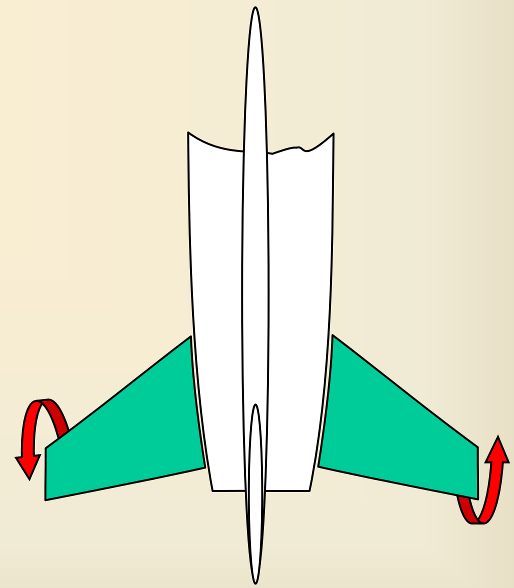
aileron



interceptor



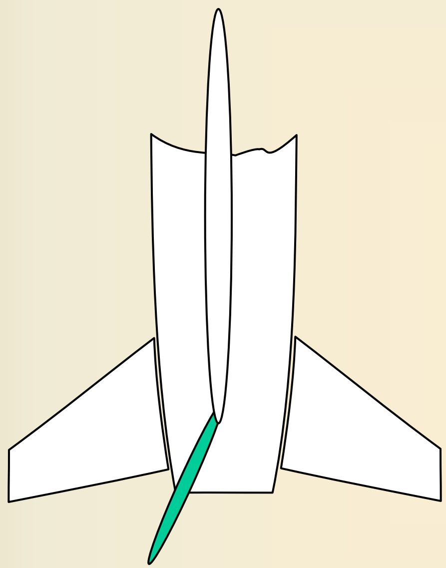
flapperon



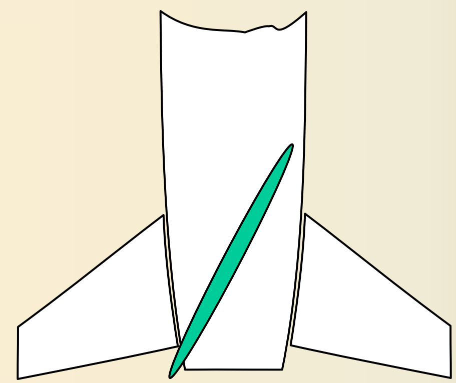
stabilizer

In directional channel

Yaw control

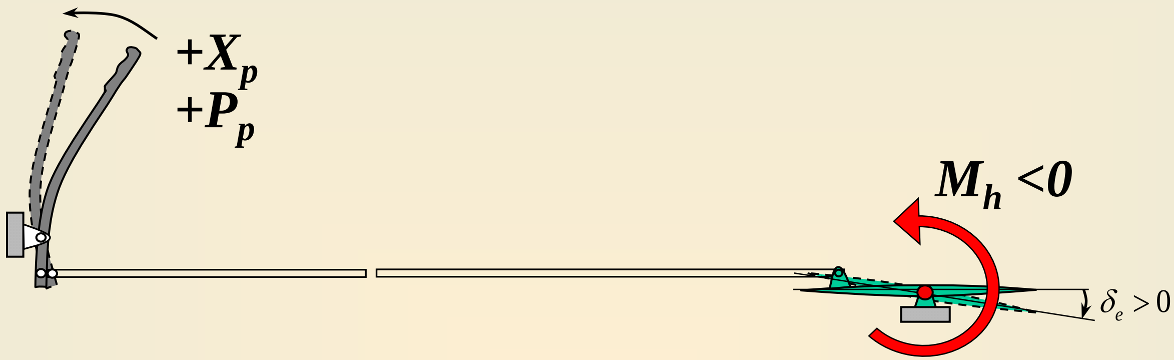


rudder



vertical tail

Control surface changes its position in result of displacement of manipulator



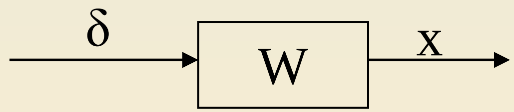
Types of manipulators:

- – central stick
- – wheel
- – side stick
- – pedals

→ for change of positions of aileron, elevator, …..

throttle lever → for change fuel amount

Aircraft dynamics (airframe) describes the relationship between the state variables x(θ, ψ ,φ…) and controls δ(δe, δa…)



It is described by the system of 12 (in general case) differential equations:

•**<u>3 equations of forces</u>** describing the relations between the applied forces and linear accelerations ($m\frac{dV}{dt} = F(x, \delta, t)$)

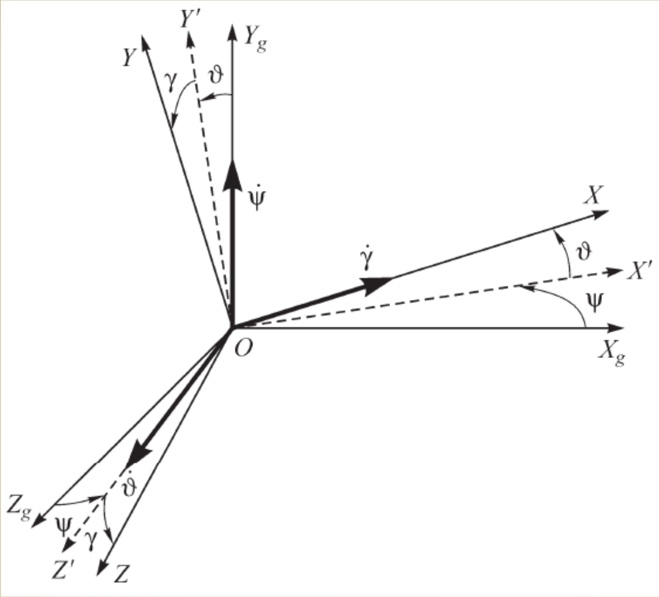
•**<u>3 equations of moments</u>** describing the relation between the moments and angular accelerations $\frac{d\overline{K}}{dt} = M(x, \delta, t); \overline{K} = I\overline{\omega}$

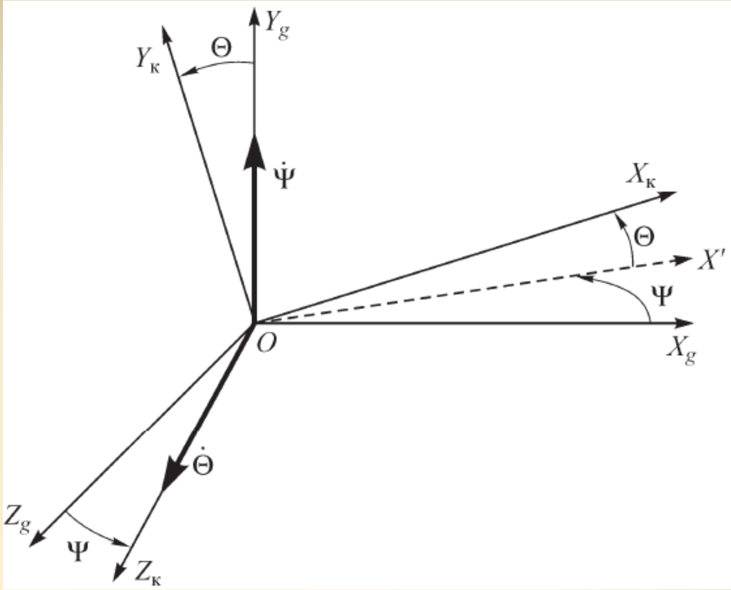
<u>+ Kinematics equations:</u>

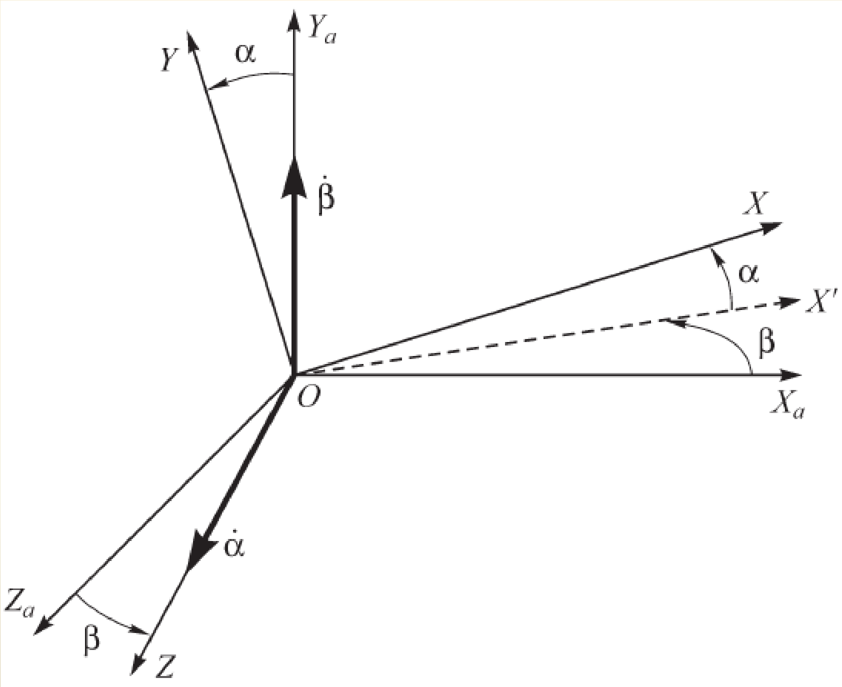
•3 Euler equations describing the relationship between the angles (θ, ψ, φ) and angular velocities (p, q, r)

•3 equations determining the relationship between the linear

displacement (h, x, y) and Euler angles (θ, ψ, φ)


Y
Y′
Y_g
γ
ϑ
$\dot{\psi}$
X
$\dot{\gamma}$
ϑ
X′
ψ
O
X_g
Z_g
$\dot{\vartheta}$
ψ
γ
Z′
Z


Y_g
$Y_к$
Θ
$\dot{\Psi}$
$X_к$
Θ
X′
Ψ
O
X_g
$\dot{\Theta}$
Z_g
Ψ
$Z_к$


Y_a
Y
α
$\dot{\beta}$
X
α
X′
β
O
X_a
$\dot{\alpha}$
Z_a
β
Z

Equation of aircraft motion

Euler equations

$$\dot{\vartheta} = \omega_y \sin\gamma + \omega_z \cos\gamma;$$

$$\dot{\gamma} = \omega_x - \left(\omega_y \cos\gamma - \omega_z \sin\gamma\right) \cdot tg\,\vartheta;$$

$$\dot{\psi} = \frac{1}{\cos\vartheta}\left(\omega_y \cos\gamma - \omega_z \sin\gamma\right).$$

Equations of moments

$$I_x \frac{d\omega_x}{dt} = (I_y - I_z)\omega_y \omega_z + M_{R_x};$$

$$I_y \frac{d\omega_y}{dt} = (I_z - I_x)\omega_x \omega_z + M_{R_y};$$

$$I_z \frac{d\omega_z}{dt} = (I_x - I_y)\omega_x \omega_y + M_{R_z}.$$

Equations for forces

$$m(t)\frac{dV_k}{dt} = P\cos(\alpha+\phi_p)\cos\beta - X_a - [\cos\alpha\cos\beta\sin\vartheta - \left(\sin\beta\sin\gamma+\sin\alpha\cos\beta\cos\gamma\right)\cos\vartheta]mg$$

$$m(t)V_k\cos\beta\frac{d\alpha}{dt} = mV_k\left(-\omega_x\cos\alpha\sin\beta+\omega_y\sin\alpha\sin\beta+\omega_z\cos\beta\right) - P\sin(\alpha+\phi_p) - Y_a + [\sin\alpha\sin\vartheta+\cos\alpha\cos\gamma\cos\vartheta]mg$$

$$m(t)V_k\frac{d\beta}{dt} = mV_k\left(\omega_x\sin\alpha+\omega_y\cos\alpha\right) - P\cos(\alpha+\phi_p)\sin\beta + Z_a + [\cos\alpha\sin\beta\sin\vartheta+\left(\cos\beta\sin\gamma-\sin\alpha\sin\beta\cos\gamma\right)\cos\vartheta]mg$$

Equations for linear motion

$$\frac{dX_g}{dt} = [\cos\alpha\cos\beta\cos\vartheta\cos\psi - \left(\sin\gamma\sin\psi-\cos\gamma\sin\vartheta\cos\psi\right)\sin\alpha\cos\beta + \left(\cos\gamma\sin\psi+\sin\gamma\sin\vartheta\cos\psi\right)\sin\beta]V_k$$

$$\frac{dH}{dt} = [\cos\alpha\cos\beta\sin\vartheta - \left(\sin\beta\sin\gamma+\sin\alpha\cos\beta\cos\gamma\right)\cos\vartheta]V_k$$

$$\frac{dZ_g}{dt} = [-\cos\alpha\cos\beta\cos\vartheta\sin\psi - \left(\sin\gamma\cos\psi+\cos\gamma\sin\vartheta\sin\psi\right)\sin\alpha\cos\beta + \left(\cos\gamma\cos\psi-\sin\gamma\sin\vartheta\sin\psi\right)\sin\beta]V_k$$

The equations are nonlinear $\dot{X} = \varphi(x, \delta, t)$

Linearization procedure is used

$$\frac{d\Delta x_i}{dt} = \sum_i \frac{d\varphi}{dx_i}\Delta x_i + \sum_j \frac{d\varphi}{d\delta_i}\Delta\delta_i \qquad (*)$$

$if \quad \frac{d\varphi}{dx_i}; \ \frac{d\varphi}{d\delta_i} \approx const$, equation (*) becomes a linear equation with constant coefficients

$$\frac{d}{dt} = s \quad \longrightarrow \quad A(s)\cdot x(s) = B\cdot\delta(s)$$ – linear algebraic equations

$$\downarrow$$

$$\frac{x(s)}{\delta(s)} = \frac{N(s)}{D(s)}$$ – transfer function

Linearization

$$m(\dot{V}_0+\Delta\dot{V})=(P_0+\Delta P)\cos(\alpha_0+\Delta\alpha+\phi)\cos(\beta+\Delta\beta)-(X_{a_0}+\Delta X_a)-[\cos(\alpha_0+\Delta\alpha)\cos(\beta_0+\Delta\beta)\sin(\vartheta_0+\Delta\vartheta)-$$
$$-\cos(\vartheta_0+\Delta\vartheta)\sin(\beta_0+\Delta\beta)\sin(\gamma_0+\Delta\gamma)-\sin(\alpha_0+\Delta\alpha)\cos(\beta_0+\Delta\beta)\cos(\gamma_0+\Delta\gamma)\cos(\vartheta_0+\Delta\vartheta)]mg$$

$$\Delta P=P^V\Delta V+\Delta P_{ynp}$$

$$\Delta X_a=X_a^V\Delta V+X_a^\alpha\Delta\alpha+X_a^{\delta_B}\Delta\delta_B+X_a^\beta\Delta\beta$$

$$\Downarrow$$

$$m(\dot{V}_0+\Delta\dot{V})=X_0+X_0^\alpha\Delta\alpha+X_0^{\delta_B}\Delta\delta_B+X_0^P\Delta P_{ynp}+X_0^V\Delta V+X_0^\beta\Delta\beta+X_0^\gamma\Delta\gamma+X_0^\vartheta\Delta\vartheta,$$

where

$$X_0=P_0\cos(\alpha_0+\phi_0)\cos\beta_0+X_{a_0}-[\cos\alpha_0\cos\beta_0\sin\vartheta_0+\sin\beta_0\sin\gamma_0\cos\vartheta_0+\sin\alpha_0\cos\beta_0\cos\gamma_0\cos\vartheta_0]mg;$$

$$X_0^\alpha=-X_{a_0}^\alpha-P_0\sin(\alpha_0+\phi_0)\cos\beta_0+(\sin\alpha_0\cos\beta_0\sin\vartheta_0-\cos\alpha_0\cos\beta_0\cos\gamma_0\cos\vartheta_0)mg;$$

$$X_0^\beta=-P_0\cos(\alpha_0+\phi_0)\sin\beta_0-X_{a_0}^\beta-(-\cos\alpha_0\sin\beta_0\cos\gamma_0+\cos\beta_0\sin\gamma_0\cos\vartheta_0-\cos\vartheta_0\sin\alpha_0\sin\beta_0\cos\gamma_0)mg;$$

$$X_0^\vartheta=-(\cos\alpha_0\cos\beta_0\cos\vartheta_0-\sin\beta_0\sin\gamma_0\sin\vartheta_0-\sin\alpha_0\cos\beta_0\cos\gamma_0\sin\vartheta_0)mg$$

$$X_0^\gamma=-(\sin\beta_0\cos\gamma_0-\sin\alpha_0\cos\beta_0\sin\gamma_0)mg$$

$$X_0^V=P_0^V\cos(\alpha_0+\varphi_0)\cos\beta_0-X_{a_0}^V$$

$$X_0^P=P_0^{P_{ynp}}\cos(\alpha_0+\varphi_0)\cos\beta_0$$

$$X_0^{\delta_B}=-X_{a_0}^{\delta_B}$$

$$m\Delta\dot{V} = X_0^V \Delta V + X_0^\alpha \Delta\alpha + X_0^P \Delta P_{ynp} + X_0^{\delta_B} \Delta\delta_B + X_0^\beta \Delta\beta + X_0^\gamma \Delta\gamma + X_0^\vartheta \Delta\vartheta$$

$$m\Delta\dot{V}_0 = \Delta X;$$
$$mV_0(\Delta\dot{\alpha} - \Delta\omega_z) = \Delta Y;$$
$$mV_0(\Delta\dot{\beta} - \sin\alpha_0 \Delta\omega_x - \cos\alpha_0 \Delta\omega_y) = \Delta Z;$$
$$I_x \Delta\dot{\omega}_x + (I_Z - I_y)\omega_{Z0}\Delta\omega_y = \Delta M_x;$$
$$I_y \Delta\dot{\omega}_y + (I_x - I_Z)\omega_{Z0}\Delta\omega_x = \Delta M_y;$$
$$I_Z \Delta\dot{\omega}_Z = \Delta M_Z,$$

$$\Delta\dot{\vartheta} = \Delta\omega_Z;$$
$$\Delta\dot{\gamma} = \Delta\omega_x - tg\,\vartheta_0(\Delta\omega_y - \omega_{Z0}\Delta\gamma);$$
$$\Delta\dot{\psi} = \frac{1}{\cos\vartheta_0}(\Delta\omega_y - \omega_{Z0}\Delta\gamma).$$

Longitudinal motion $(\Delta\beta = \Delta\gamma = \Delta\omega_z = \Delta\omega_y = 0)$

$$m\Delta\dot{V} = \Delta X;$$
$$mV_0(\Delta\dot{\alpha} - \Delta\omega_z) = \Delta Y;$$
$$I_Z \Delta\dot{\omega}_Z = \Delta M_Z;$$
$$\Delta\dot{\vartheta} = \Delta\omega_Z.$$

Lateral motion

$$mV_0(\Delta\dot{\beta} - \sin\alpha_0\Delta\omega_x - \cos\alpha_0\Delta\omega_y) = \Delta Z;$$

$$I_x\Delta\dot{\omega}_x - I_{xy}\Delta\dot{\omega}_y + (I_Z - I_y)\omega_{Z0}\Delta\omega_y = \Delta M_x;$$

$$I_y\Delta\dot{\omega}_y - I_{xy}\Delta\dot{\omega}_x + (I_x - I_Z)\omega_{Z0}\Delta\omega_x = \Delta M_y;$$

$$\Delta\dot{\gamma} = \Delta\omega_x - tg\,\vartheta_0(\Delta\omega_y - \omega_{Z0}\Delta\gamma).$$

Linearized equations for linear motion + 1 Euler equation

$$\frac{d\Delta H}{dt} = \Delta V\sin\theta_0 - V\cos\theta_0(\Delta\vartheta - \Delta\alpha);$$

$$\frac{d\Delta X_g}{dt} = \Delta V\cos\theta_0 - V\cos\theta_0\cos\psi_0(\Delta\vartheta - \Delta\alpha);$$

$$\frac{d\Delta Z_g}{dt} = -V_0\cos\theta_0(\Delta\psi - \Delta\beta);$$

These equations can be calculated separately from the other

$$\Delta\dot{\psi} = \sec\vartheta_0(\Delta\omega_y - \omega_{Z0}\Delta\gamma).$$

Linearized equations for the longitudinal motion

$$\Delta\dot{V}_K = \overline{X}^V\Delta V + \overline{X}^\alpha\Delta\alpha + \overline{X}^\vartheta\Delta\vartheta + \overline{X}^{\delta_B}\Delta\delta_B + \overline{X}^P\Delta P_{ynp};$$

$$\Delta\dot{\alpha} = \Delta\omega_Z - \overline{Y}^V\Delta V - \overline{Y}^\alpha\Delta\alpha - \overline{Y}^{\delta_B}\Delta\delta_B - \overline{Y}^P\Delta P_{ynp} + \frac{g}{V}\sin\theta_0\Delta\vartheta;$$

$$\Delta\dot{\omega}_Z = \overline{M}_Z^\alpha\Delta\alpha + \overline{M}_Z^{\omega_Z}\Delta\omega_Z + \overline{M}_Z^{\dot{\alpha}}\Delta\dot{\alpha} + \overline{M}_Z^{\delta_B}\Delta\delta_B;$$

$$\Delta\dot{\vartheta} = \Delta\omega_Z.$$

Linearized equations for the lateral motion

$$\Delta\dot{\beta} = \sin\alpha_0\Delta\omega_x + \cos\alpha_0\Delta\omega_y + \bar{Z}^{\beta}\Delta\beta + \frac{g}{V_0}\cos\vartheta_0\Delta\gamma + \bar{Z}^{\delta_H}\Delta\delta_H + \bar{Z}^{\delta_Э}\Delta\delta_Э;$$

$$\Delta\dot{\omega}_x = \bar{M}_{x_0}^{\beta}\Delta\beta + \bar{M}_{x_0}^{\dot{\beta}}\Delta\dot{\beta} + \bar{M}_{x_0}^{\omega_x}\Delta\omega_x + \bar{M}_{x_0}^{\omega_y}\Delta\omega_y + \bar{M}_{x_0}^{\delta_Э}\Delta\delta_Э + \bar{M}_{x_0}^{\delta_H}\Delta\delta_H;$$

$$\Delta\dot{\omega}_y = \bar{M}_{y_0}^{\beta}\Delta\beta + \bar{M}_{y_0}^{\dot{\beta}}\Delta\dot{\beta} + \bar{M}_{y_0}^{\omega_y}\Delta\omega_y + \bar{M}_{y_0}^{\omega_x}\Delta\omega_x + \bar{M}_{y_0}^{\delta_Э}\Delta\delta_Э + \bar{M}_{y_0}^{\delta_H}\Delta\delta_H;$$

$$\Delta\dot{\gamma} = \Delta\omega_x - tg\,\vartheta_0\Delta\omega_y.$$

Longitudinal motion

The equation in Laplace transform

$$(P-\bar{X}^V)V(s) - \bar{X}^{\alpha}\alpha(s) - \bar{X}^{\vartheta}\vartheta(s) = \bar{X}^{\delta_в}\delta_в(s) + \Delta\bar{P} - sW_{x_g}(s)$$

$p-Laplace\ operator$

$p=s$

$$\bar{Y}^V V(s) + (P+\bar{Y}^{\alpha})\alpha(s) - \omega_Z(s) = -\bar{Y}^{\delta_в}\delta_в(s) + s\alpha_T(s)$$

$$-\bar{M}_Z^V V(s) - (\bar{M}_Z^{\alpha} + s\bar{M}_Z^{\dot{\alpha}})\alpha(s) + (s-\bar{M}_Z^{\omega_Z})\omega_Z(s) = \bar{M}_Z^{\delta_в}\delta_в(s)$$

$$s\vartheta(s) = \omega_Z(s)$$

$$A(s)x(s) = B(s)u(s) + E(s)W(s)$$

$$A = \begin{vmatrix} (P-\bar{X}^V) & -\bar{X}^{\alpha} & 0 & g \\ \bar{Y}^V & (P+\bar{Y}^{\alpha}) & -1 & 0 \\ -\bar{M}_Z^V & -(s\bar{M}_Z^{\dot{\alpha}} + \bar{M}_Z^{\alpha}) & (s-\bar{M}_Z^{\omega_Z}) & 0 \\ 0 & 0 & -1 & s \end{vmatrix}; \quad x(s) = \begin{vmatrix} V(s) \\ \alpha(s) \\ \omega_Z(s) \\ \vartheta(s) \end{vmatrix} \quad B = \begin{vmatrix} \bar{X}^{\delta_e} & 1 \\ \bar{Y}^{\delta_e} & 0 \\ \bar{M}_Z^{\delta_e} & 0 \\ 0 & 0 \end{vmatrix};$$

$$u(s) = \begin{vmatrix} \delta_в \\ \Delta\bar{P} \end{vmatrix} \quad E = \begin{vmatrix} 0 & -s \\ s & 0 \end{vmatrix} \quad W = \begin{vmatrix} \alpha_T \\ W_{X_g} \end{vmatrix}$$

$$\Delta = s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4$$

Transfer functions

$$W_{y_i}^{\alpha}(s)=\frac{\Delta_{y_i}^{\alpha}(s)}{\Delta(s)} \qquad W_{y_i}^{\vartheta}(s)=\frac{\Delta_{y_i}^{\vartheta}(s)}{\Delta(s)} \qquad W_{y_i}^{V}(s)=\frac{\Delta_{y_i}^{V}(s)}{\Delta(s)}$$

$$\Delta_{\delta_e}^{V}=\begin{vmatrix} 0 & -\bar{X}^{\alpha} & 0 & g \\ \bar{Y}^{\delta_e} & (P+\bar{Y}^{\alpha}) & -1 & 0 \\ \bar{M}_Z^{\delta_e} & -(s\bar{M}_Z^{\dot{\alpha}}+\bar{M}_Z^{\alpha}) & (s-\bar{M}_Z^{\omega_Z}) & 0 \\ 0 & 0 & -1 & s \end{vmatrix}$$

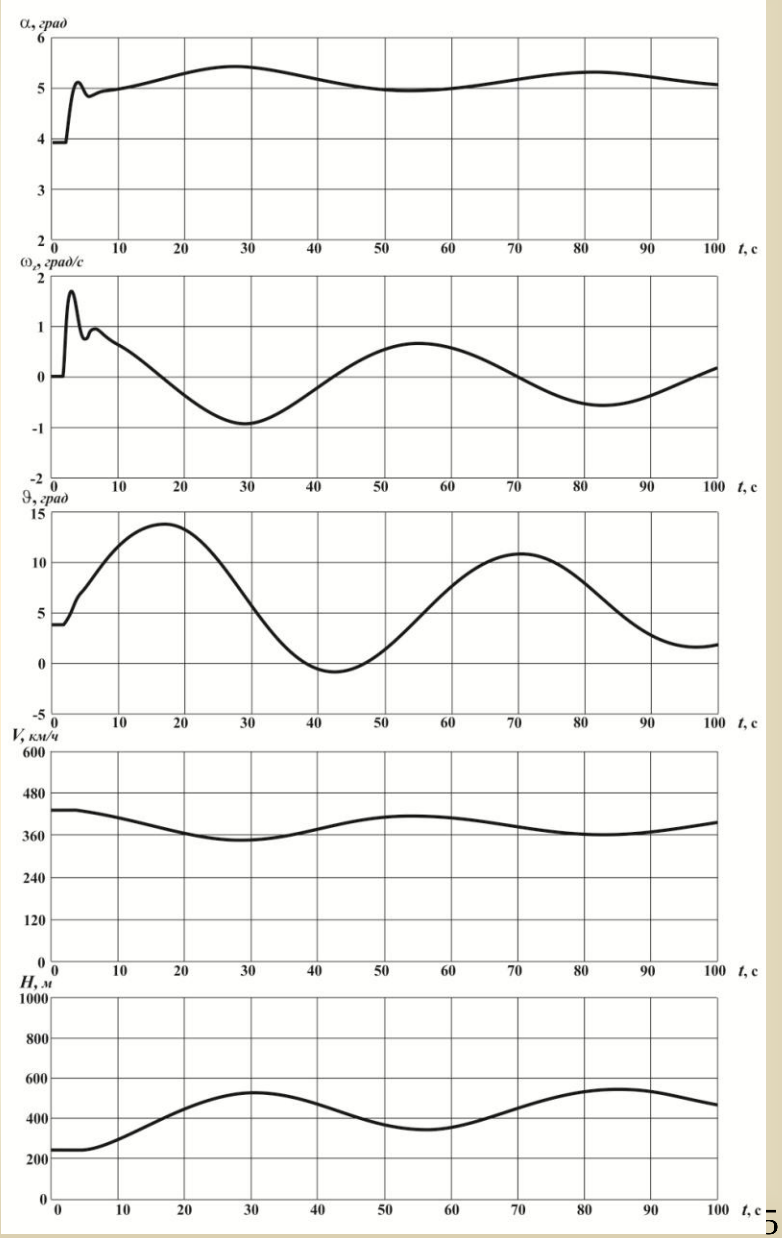
	K(p)	***B*₁**	***B*₂**	***B*₃**	***B*₄**
$\Delta_{\delta_e}^{V}$	1	0	$-\bar{X}^{\alpha}\bar{Y}^{\delta_e}$	$(\bar{X}^{\alpha}\bar{Y}^{\delta_e}\bar{M}_Z^{\omega_Z}-\bar{X}^{\alpha}\bar{M}_Z^{\delta_e}+$ $+g\bar{Y}^{\delta_e}\bar{M}_Z^{\dot{\alpha}}-g\bar{M}_Z^{\delta_e})$	$g(\bar{M}_Z^{\alpha}\bar{Y}^{\delta_e}-\bar{M}_Z^{\delta_e}\bar{Y}^{\alpha})$
$\Delta_{\delta_e}^{\alpha}$	1	$-\bar{Y}^{\delta_e}$	$\bar{Y}^{\delta_в}\bar{M}_Z^{\omega_Z}+$ $+\bar{X}^{V}\bar{Y}^{\delta_e}+\bar{M}_Z^{\delta_e}$	$-\bar{Y}^{\delta_e}\bar{X}^{V}\bar{M}_Z^{\omega_Z}-\bar{X}^{V}\bar{M}_Z^{\delta_e}$	$g(\bar{Y}^{V}\bar{M}_Z^{\delta_e}-\bar{M}_Z^{V}\bar{Y}^{\delta_e})$
$\Delta_{\delta_e}^{\vartheta}$	1	0	$\bar{M}_Z^{\delta_в}-\bar{M}_Z^{\dot{\alpha}}\bar{Y}^{\delta_в}$	$\bar{M}_Z^{\delta_e}(\bar{Y}^{\alpha}-\bar{X}^{V})+$ $+\bar{Y}^{\delta_e}(\bar{X}^{V}\bar{M}_Z^{\dot{\alpha}}-\bar{M}_Z^{\alpha})$	$\bar{M}_Z^{\delta_e}(\bar{Y}^{V}\bar{X}^{\alpha}-\bar{X}^{V}\bar{Y}^{\alpha})+$ $+\bar{Y}^{\delta_e}(\bar{X}^{V}\bar{M}_Z^{\alpha}-\bar{X}^{\alpha}\bar{M}_Z^{V})$
$\Delta_{\delta_e}^{n_y}$	$\frac{V}{g}s$	$\bar{Y}^{\delta_e}$	$\bar{Y}^{\delta_e}(-\bar{X}^{V}-\bar{M}_Z^{\omega_Z}-$ $-\bar{M}_Z^{\dot{\alpha}})$	$\bar{M}_Z^{\delta_e}\bar{Y}^{\alpha}+\bar{Y}^{\delta_e}[\bar{X}^{V}(\bar{M}_Z^{\dot{\alpha}}+$ $+\bar{M}_Z^{\omega_Z})-\bar{M}_Z^{\alpha}]$	$\bar{M}_Z^{\delta_e}(\bar{Y}^{V}\bar{X}^{\alpha}-\bar{X}^{V}\bar{Y}^{\alpha}-$ $-g\bar{Y}^{V})+\bar{Y}^{\delta_e}(\bar{X}^{V}\bar{M}_Z^{\alpha}-$ $-\bar{X}^{\alpha}\bar{M}_Z^{V}+g\bar{M}_Z^{V})$
$\Delta=s^4+a_1s^3+a_2s^2+a_3s+a_4$					
		***a*₁**	***a*₂**	***a*₃**	***a*₄**
		$\bar{Y}^{\alpha}-\bar{M}_Z^{\omega_Z}-$ $-\bar{M}_Z^{\dot{\alpha}}-\bar{X}^{V}$	$-\bar{M}_Z^{\omega_Z}\bar{Y}^{\alpha}-\bar{M}_Z^{\alpha}-$ $-\bar{X}^{V}\bar{Y}^{\alpha}+\bar{X}^{\alpha}\bar{Y}^{V}+$ $+\bar{X}^{V}(\bar{M}_Z^{\omega_Z}+\bar{M}_Z^{\dot{\alpha}})$	$\bar{X}^{V}\bar{Y}^{\alpha}\bar{M}_Z^{\omega_Z}+\bar{X}^{V}\bar{M}_Z^{\alpha}-$ $-\bar{X}^{\alpha}\bar{Y}^{V}\bar{M}_Z^{\omega_Z}-\bar{X}^{\alpha}\bar{M}_Z^{V}+$ $+g\bar{M}_Z^{V}-g\bar{Y}^{V}\bar{M}_Z^{\dot{\alpha}}$	$\bar{M}_Z^{V}\bar{Y}^{\alpha}g-g\bar{Y}^{V}\bar{M}_Z^{\alpha}$

Division of motion on short period motion and path motion

$$\Delta = (s^2 + 2\xi_k \omega_k s + \omega_k^2)(s^2 + 2\xi_d \omega_d s + \omega_d^2)$$

$$\omega_k >> \omega_d$$

$$\xi_k >> \xi_d$$



Short period motion

$$(s+\bar{Y}^{\alpha})\alpha(s)-\omega_Z(s)=-\bar{Y}^{\delta_в}\delta_в+s\alpha_T$$

$$-(\bar{M}_Z^{\alpha}+\bar{M}_Z^{\dot{\alpha}}s)\alpha(s)+(s-\bar{M}_Z^{\omega_Z})\omega_Z=\bar{M}_Z^{\delta_в}\delta_в$$

$$s\vartheta=\omega_Z$$

W	***Transfer function***	***Simplified equation***
$\frac{\alpha(s)}{\delta_в(s)}$	$\frac{-\bar{Y}^{\delta_в}s+\bar{M}_Z^{\delta_в}+\bar{Y}^{\delta_в}\bar{M}_Z^{\omega_Z}}{\Delta}$	$\frac{\bar{M}_Z^{\delta_в}}{\Delta}$
$\frac{\vartheta(s)}{\delta_в(s)}$	$\bar{M}_Z^{\delta_в}\left[\frac{s\left(1-\frac{\bar{Y}^{\delta_в}\bar{M}_Z^{\dot{\alpha}}}{\bar{M}_Z^{\delta_в}}\right)+\left(\bar{Y}^{\alpha}-\frac{\bar{Y}^{\delta_в}\bar{M}_Z^{\alpha}}{\bar{M}_Z^{\delta_в}}\right)}{s\Delta}\right]$	$\frac{\bar{M}_Z^{\delta_в}(s+\bar{Y}^{\alpha})}{s\Delta}$
$\frac{n_y(s)}{\delta_в(s)}$	$\frac{\frac{V}{g}\left[\bar{Y}^{\delta_в}s^2+\bar{Y}^{\delta_в}(-\bar{M}_Z^{\omega_Z}-\bar{M}_Z^{\dot{\alpha}})s+\left(\bar{Y}^{\alpha}\bar{M}_Z^{\delta_в}-\bar{Y}^{\delta_в}\bar{M}_Z^{\alpha}\right)\right]}{\Delta}$	$\frac{\bar{M}_Z^{\delta_в}\bar{Y}^{\alpha}}{\Delta}\frac{V}{g}$
$\frac{\theta(s)}{\delta_e(s)}$	$\frac{\bar{Y}^{\delta_в}s^2+\bar{Y}^{\delta_в}(-\bar{M}_Z^{\omega_Z}-\bar{M}_Z^{\dot{\alpha}})s+\bar{Y}^{\alpha}\bar{M}_Z^{\delta_в}-\bar{Y}^{\delta_в}\bar{M}_Z^{\alpha}}{s\Delta}$	$\frac{\bar{M}_Z^{\delta_в}\bar{Y}^{\alpha}}{s(s^2+2\xi_k\omega_k s+\omega_k^2)}$

W	*Transfer function*
$\frac{\alpha(s)}{\alpha_T(s)}$	$\frac{s(s-\bar{M}_Z^{\omega_Z})}{\Delta}$
$\frac{\vartheta(s)}{\alpha_T}$	$\frac{\bar{M}_Z^{\alpha}}{\Delta}$
$\frac{n_y}{\alpha_T}=s\frac{V}{g}\frac{\theta(s)}{\alpha_T}$	$\frac{\frac{V}{g}s\left[\left(-\bar{M}_Z^{\dot{\alpha}}+\bar{Y}^{\alpha}\right)s-\bar{M}_Z^{\omega_Z}\bar{Y}^{\alpha}\right]}{\Delta}$
$\frac{\theta(s)}{\alpha_T}$	$\frac{(-\bar{M}_Z^{\dot{\alpha}}+\bar{Y}^{\alpha})s-\bar{M}_Z^{\omega_Z}\bar{Y}^{\alpha}}{\Delta}$

$$\Delta(s)=s^2+2\xi_k\omega_k s+\omega_k^2$$

$$\omega_k^2=-\bar{M}_Z^{\alpha}-\bar{M}_Z^{\omega_Z}\bar{Y}^{\alpha}$$

$$2\xi_k\omega_k=-\bar{M}_Z^{\omega_Z}-\bar{M}_Z^{\dot{\alpha}}+\bar{Y}^{\alpha}$$

$$\xi_k=\frac{-\bar{M}_Z^{\omega_Z}-\bar{M}_Z^{\dot{\alpha}}\bar{Y}^{\alpha}}{2\sqrt{-\bar{M}_Z^{\alpha}-\bar{M}_Z^{\omega_Z}\bar{Y}^{\alpha}}}$$

$$\omega_k^2=-\frac{C_y^{\alpha}qSb_a}{I_Z}\sigma_n$$

$$\xi_k,\ \omega_k=f(M,H)$$

$$\sigma_n=m_z^{C_y}+\frac{m_z^{\bar{\omega}_z}}{\mu}$$

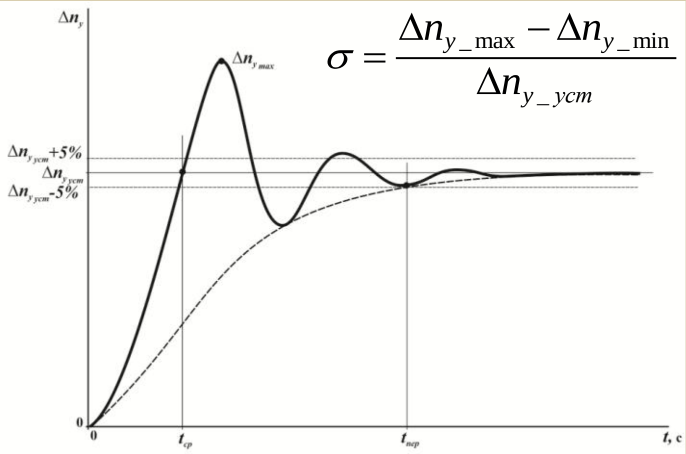
where $\mu=\frac{2m}{\rho Sb_a}$

Time responses

$$\Delta n_{ya} = \Delta n_{ya\,ycm}\left(1 - \frac{e^{-\xi_\kappa \omega_\kappa t}\cdot \sin\sqrt{1-\xi_\kappa^2}\,\omega_\kappa t + \varphi}{\sqrt{1-\xi_\kappa^2}}\right)$$

where $\varphi = \arcsin\sqrt{1-\xi^2};\quad \Delta n_{y\_ycm} = \dfrac{n_y^\alpha\cdot\Delta X + \bar{M}_z^\delta K_ш}{\omega_k^2}$

$$t_{cp} = \frac{\pi - \arcsin\sqrt{1-\xi_k^2}}{\omega_k\sqrt{1-\xi_k^2}};\qquad t_{nep} \approx \frac{3}{\xi_k\omega_k};\qquad \sigma_{\Delta n_y} = e^{-\frac{\ln \xi_\kappa}{\sqrt{1-\xi_\kappa^2}}}$$



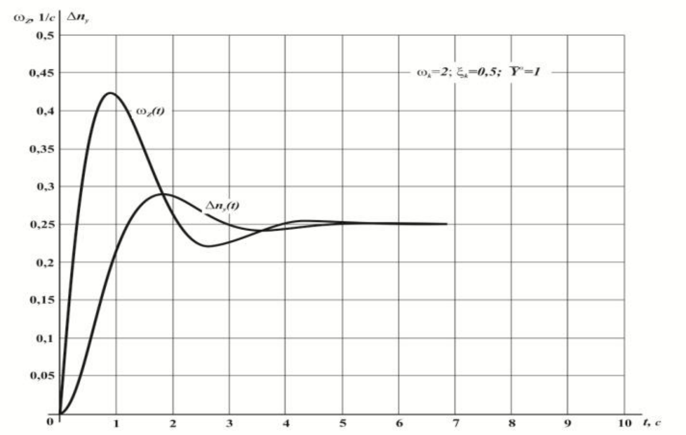
$$\sigma = \frac{\Delta n_{y\_\max} - \Delta n_{y\_\min}}{\Delta n_{y\_ycm}}$$

$$\frac{\omega_Z(s)}{X_B(s)} = \frac{\bar{M}_Z^{\delta_e} K_Ш (s + \bar{Y}_a^\alpha)}{s^2 + 2\xi_\kappa\omega_\kappa s + \omega_\kappa^2}$$

$$\dot{\omega}_z\big|_{t\to 0} = \Delta X\cdot \bar{M}_z^\delta \cdot K_ш$$

characteristics $\omega_k,\ \xi_k,\ n_y^\alpha$

$$\frac{\dot{\omega}_z\big|_{t=0}}{n_y(t\Rightarrow\infty)} = \frac{\omega_\kappa^2}{n_y^\alpha}$$



Static characteristics

$$\frac{\Delta n_{y_a}}{\Delta\delta_в} = \frac{n_y^\alpha \bar{M}_Z^{\delta_в}}{\omega_k^2} = \frac{1}{C_{y\ hor.fl.}}\frac{\bar{M}_Z^{\delta_в}}{\sigma_n}$$

$$X^{n_y} = \frac{\sigma_n C_{y\ hor.fl.}}{m_z^\delta K_ш}$$

$$\frac{\delta_n}{n_{y_a}} = \delta^{n_y} = \frac{-\sigma_n}{m_z^{\delta_в}} C_{y\ hor.fl.}$$

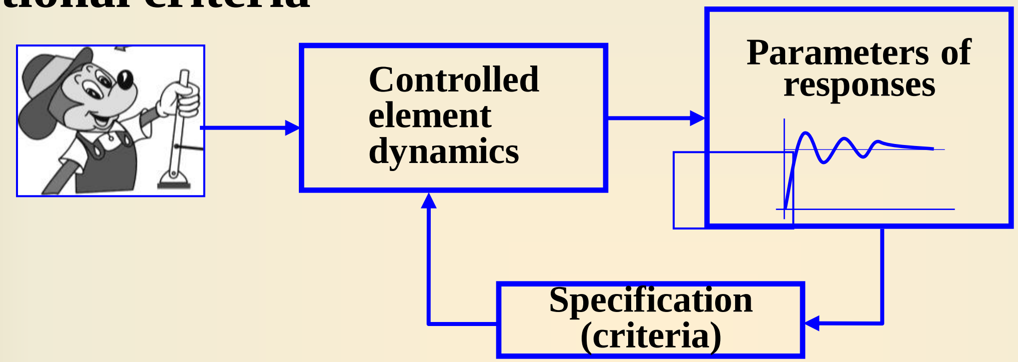
The dynamic and static characteristics (handing qualities (flying qualities)) change broadly in H, V range

Criteria used for pilot-vehicle system design - flying qualities criteria

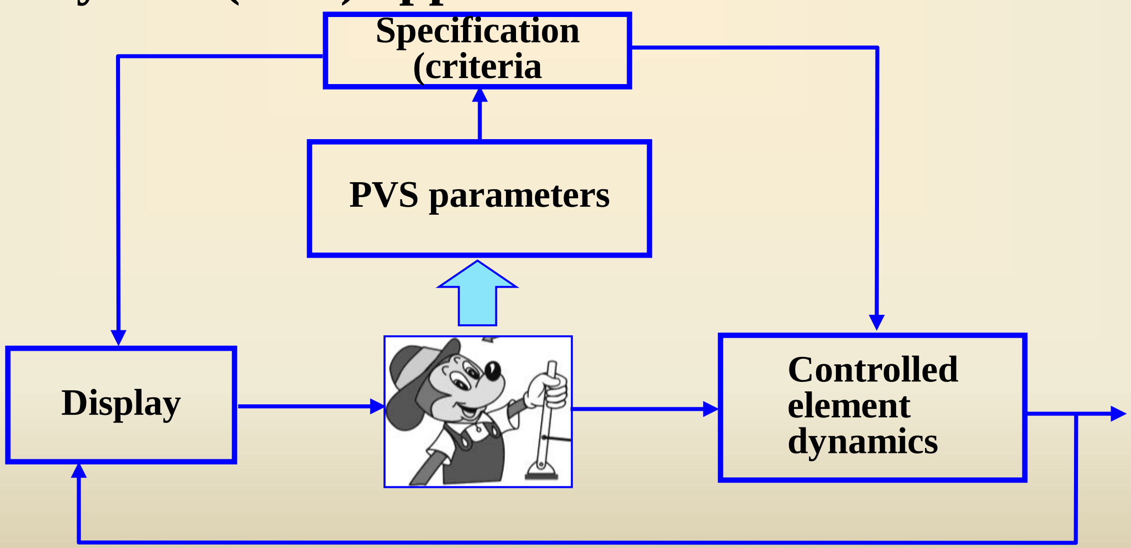
«Flying qualities of an aircraft are those properties which describe the ease and effectiveness with which it responds to pilot commands in the execution of some flight task».

D. McRuer
M. Cock

1. Traditional criteria



2. Pilot–vehicle system (PVS) approach



Criteria – are the requirements to the FQ

Accepted principle in specification

- Davison of requirements on the class of aircraft

Class I **Maneuverable aircraft** $(n_y \geq 7)$

Class II **Aircraft with limited maneuverability n_y=3.5÷5** $(m < 50 \div 60\ ton)$

Class III **Non-maneuverable aircraft**

IIIa – $n_y < 3.5$

IIIb – heavy aircraft with weight > 100 T

Phase of flight: **A – precise tracking tasks, maneuvering tasks;**

B – take-off and landing tasks;

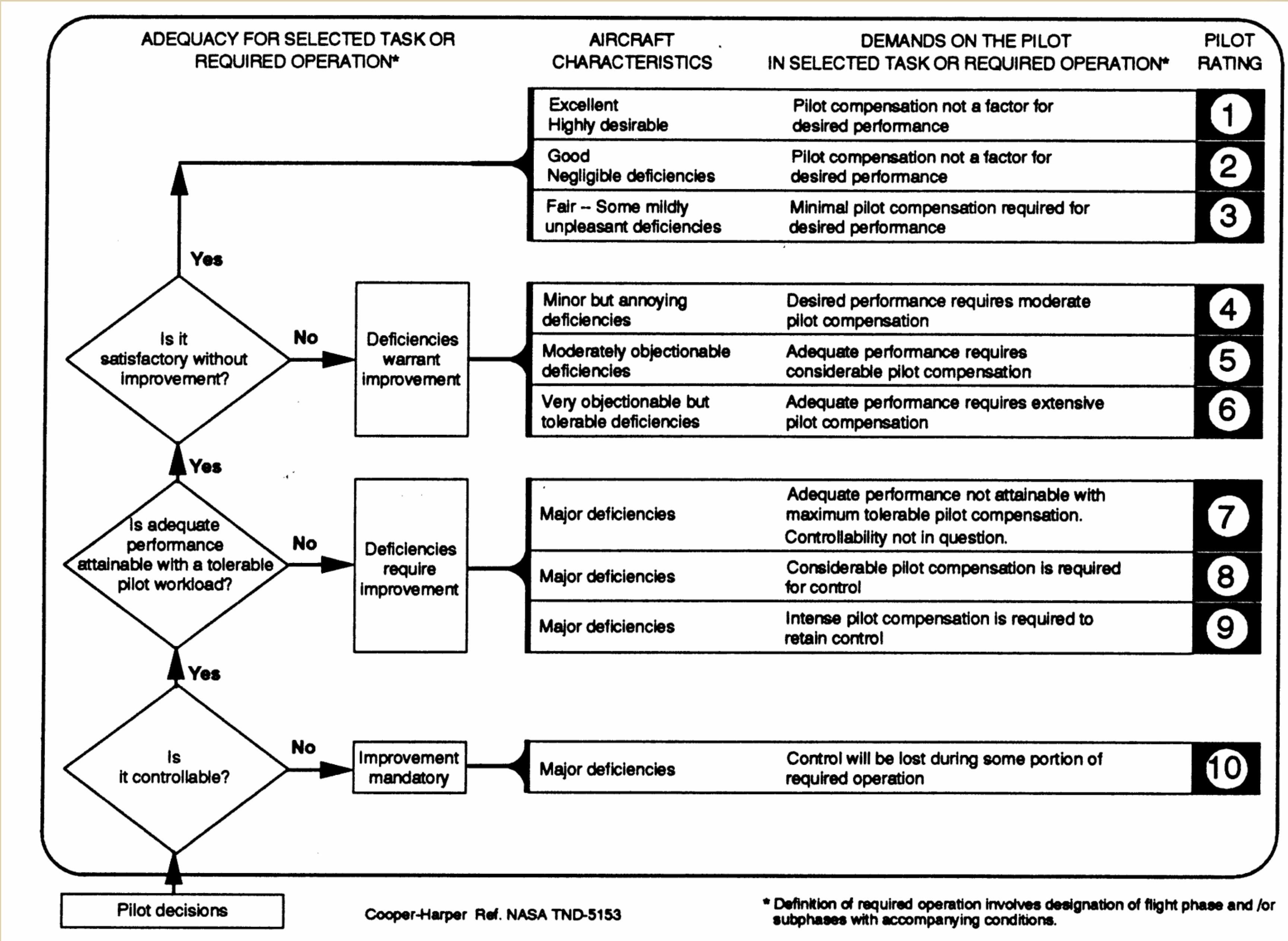
C – tasks which do not require precise control.

Level pilot rating: **level 1 – satisfactory FQ**

level 2 – acceptable FQ

level 3 – unsatisfactory FQ

Cooper-Harper rating scale



Requirements to static handling qualities

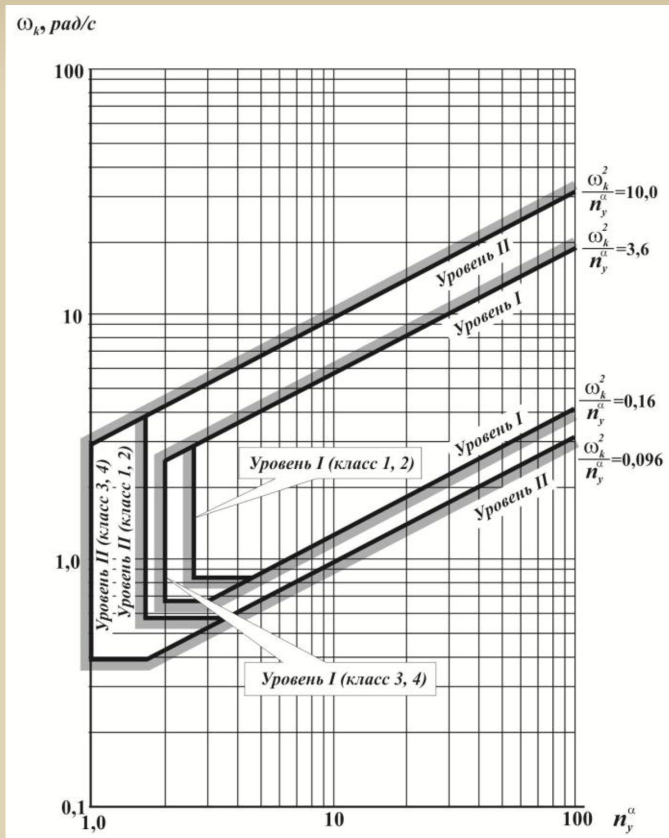
Aircraft class	I	II	III	
			a	б
$X_{\min}^{n_y}, \left[\frac{\text{мм}}{\text{ед.пер}}\right]$	-10	-20	-30	-45
$P_{\min}^{n_y}, \left[\frac{\text{н}}{\text{ед.пер}}\right]$	-10 …-30	-30 … -100	-100 … -300	-150 … -450

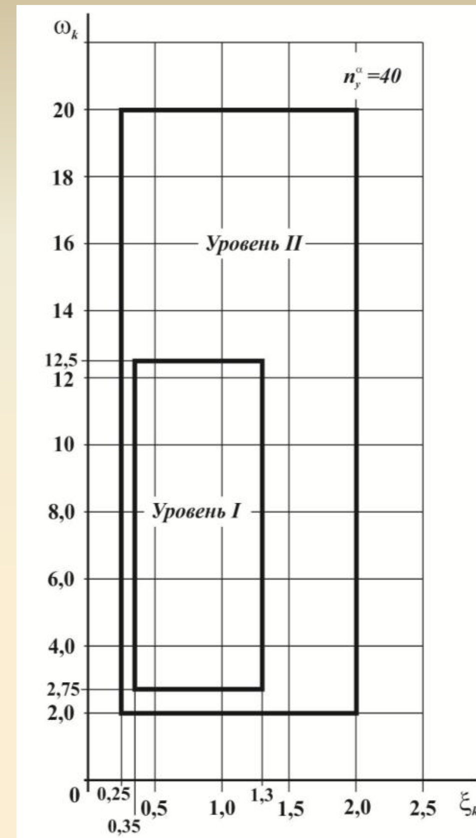
$$\frac{\left|X_{B\max}^{n_y}\right|}{\left|X_{B\min}^{n_y}\right|} \leq 3$$

Requirements to dynamic handling qualities

	I, II	III a	III b
A	< 0.15	< 0.2	< 0.3
B	< 0.25	< 0.3	< 0.3
C	< 0.25	< 0.35	< 0.4

$\left.\right] \sigma_{n_y}$





Уровни оценок	Категория А		Категория Б	
	$\frac{\omega_\kappa^2}{n_y^\alpha}_{\min}$ [1/с^2]	$\frac{\omega_\kappa^2}{n_y^\alpha}_{\max}$ [1/с^2]	$\frac{\omega_\kappa^2}{n_y^\alpha}_{\min}$ [1/с^2]	$\frac{\omega_\kappa^2}{n_y^\alpha}_{\max}$ [1/с^2]
I	0,28	3,6	0,16	3,6
II	0,16	10	0,096	10

Уровень оценки	Категории А и Б	
	$\xi_{\kappa\,\min}$	$\xi_{\kappa\,\max}$
1	0,35	1,3
2	0,25	2,0

Path motion

$$(p-\bar{X}^V)V(p)-\bar{X}^\alpha\alpha(p)+g\vartheta(p)=\Delta\bar{P}-pW_x(p); \qquad -M_Z^V V(p)-M_Z^\alpha\alpha(p)=M_Z^{\delta_в}\delta_в(p);$$

$$\bar{Y}^V V(p)+(p+\bar{Y}^\alpha)\alpha(p)-p\vartheta(p)=-\bar{Y}^{\delta_в}\delta_в(p)+p\alpha_T;$$

	Точное выражение $W(p)$	Упрощенное выражение*) $W(p)$
$\frac{V(s)}{\delta_e(s)}$	$\frac{\bar{M}_Z^{\delta_e}(\bar{X}^\alpha-g)\left[s-\frac{g}{(\bar{X}^\alpha-g)}\left(\bar{Y}^\alpha-\bar{Y}^{\delta_e}\frac{\bar{M}_Z^\alpha}{\bar{M}_Z^{\delta_в}}\right)\right]}{\Delta}$	$\frac{\bar{M}_Z^{\delta_в}(\bar{X}^\alpha-g)\left(s-\frac{g}{(\bar{X}^\alpha-g)}\bar{Y}^\alpha\right)}{\Delta}$
$\frac{\alpha(s)}{\delta_e(s)}$	$\frac{\bar{M}_Z^{\delta_в}\left[s^2+s(-\bar{X}^V)\right]}{\Delta}+\frac{\bar{M}_Z^{\delta_e}g\left(\bar{Y}^V-\bar{Y}^{\delta_в}\frac{\bar{M}_Z^V}{\bar{M}_Z^{\delta_в}}\right)}{\Delta}$	$\frac{\bar{M}_Z^{\delta_в}\left[s^2+s(-\bar{X}^V)+g\bar{Y}^V\right]}{\Delta}$
$\frac{\vartheta(s)}{\delta_e(s)}$	$\frac{\bar{M}_Z^{\delta_в}\left[s^2+s\left(\bar{Y}^\alpha-\bar{X}^V-\bar{M}_Z^\alpha\frac{\bar{Y}^{\delta_в}}{\bar{M}_Z^{\delta_в}}\right)\right]}{\Delta}+\frac{\bar{M}_Z^{\delta_в}\left(\bar{X}^\alpha\bar{Y}^V-\bar{X}^V\bar{Y}^\alpha-\bar{X}^\alpha\frac{\bar{M}_Z^V\bar{Y}^{\delta_в}}{\bar{M}_Z^{\delta_в}}+\frac{\bar{M}_Z^\alpha\bar{X}^V\bar{Y}^{\delta_в}}{\bar{M}_Z^{\delta_в}}\right)}{\Delta}$	$\frac{\bar{M}_Z^{\delta_в}\left[s^2+s(\bar{Y}^\alpha-\bar{X}^V)+\bar{X}^\alpha\bar{Y}^V-\bar{X}^V\bar{Y}^\alpha\right]}{\Delta}$
$\frac{\Theta(s)}{\delta_e(s)}$	$\frac{\bar{M}_Z^{\delta_в}s\left[\bar{Y}^\alpha-\frac{\bar{M}_Z^\alpha\bar{Y}^{\delta_в}}{\bar{M}_Z^{\delta_в}}\right]}{\Delta}+\frac{\bar{M}_Z^{\delta_в}\left[-\bar{X}^V\bar{Y}^\alpha+\left(\bar{X}^\alpha-g\right)\bar{Y}^V-\left(\bar{X}^\alpha-g\right)\frac{\bar{M}_Z^V}{\bar{M}_Z^{\delta_в}}\bar{Y}^{\delta_в}+\frac{\bar{M}_Z^\alpha\bar{X}^V\bar{Y}^{\delta_в}}{\bar{M}_Z^{\delta_в}}\right]}{\Delta}$	$\frac{\bar{M}_Z^{\delta_В}\left[s\bar{Y}^\alpha+\left(-\bar{X}^V\bar{Y}^\alpha+\bar{Y}^V\left(\bar{X}^\alpha-g\right)\right)\right]}{\Delta}$
$\Delta=\omega_{k_*}^2\left[s^2+s\left(-\bar{X}^V-(\bar{X}^\alpha-g)\frac{\bar{M}_Z^V}{\omega_{k_*}^2}+(\bar{X}^V\bar{Y}^\alpha-\bar{X}^\alpha\bar{Y}^V)\frac{\bar{M}_Z^{\omega_Z}}{\omega_{k_*}^2}\right)+g\left(\frac{-\bar{Y}^V\bar{M}_Z^\alpha+\bar{Y}^\alpha\bar{M}_Z^V}{\omega_{k_*}^2}\right)\right]\cong\omega_{k_*}^2\left[s^2+2\xi_\partial\omega_\partial s+\omega_\partial^2\right]$		

If we will suppose that $\Delta\alpha=0$ **then** $\bar{Y}^V V(p)-p\vartheta(p)=-\bar{Y}^{\delta_в}\delta_в(p);$

For small $\bar{X}^V$ $\quad (p-\bar{X}^V)\Delta V(p)+g\vartheta(p)=0$ **will be** $V\cdot\Delta\dot{V}+g\cdot V\cdot\Delta\vartheta\cdot\Delta H=0 \;\Vert\!\Rightarrow \frac{V^2}{2}+gH=const$

$$\omega_\kappa^2 = -\bar{M}_Z^\alpha$$

$$2\xi_\partial \omega_\partial \cong 2h_Д \cong -\bar{X}^V - \bar{X}^\alpha \frac{\bar{M}_Z^V}{\omega_k^2}$$

$$\omega_\partial^2 = 2\frac{g^2}{V^2 m_z^{C_y}}\left[m_Z^{C_y}\left(1+\frac{C_y^V V}{2C_{y_{h.f.}}}\right) - \frac{V}{2C_{y_{h.f.}}} m_Z^{V^*}\right]$$

$$\sigma_V = \left[m_Z^{C_y}\left(1+\frac{C_y^V V}{2C_{y_{h.f.}}}\right) - \frac{V}{2C_{y_{h.f.}}} m_Z^{V^*}\right]$$

$\sigma_V < 0$ $\qquad$ $\omega_\partial^2 > 0$

$$\sigma_V = m_z^{C_y} - \frac{V}{2} C_{y_{h.f.}} m_z^V$$

when $m_z^V = 0 \rightarrow$ $\omega_\partial^2 = 2\dfrac{g^2}{V^2}$

when $\bar{P}^V \cong 0 \rightarrow$ $-\bar{X}^V = \dfrac{2C_x}{C_{y_{h.f.}} V} = \dfrac{2}{V}\dfrac{g}{K_{h.f.}}$

$$\xi_\partial = \frac{1}{K_{h.f.}\sqrt{2}}$$

Static handling qualities characteristics

From the transfer function $\frac{\Delta V(s)}{\Delta \delta_в(s)}$

$$\frac{\Delta \delta_e}{\Delta V} = 2\frac{\sigma_V}{\sigma_n}\frac{g^2}{V^2}\omega_k^2$$

$$\delta^V = \frac{\sigma_V}{m_Z^{\delta_e}}\frac{2C_{y_{h.f.}}}{V}$$

For the speed stable aircraft $\sigma_V < 0$

$\sigma_V > 0$

$X^V > 0$ X^V, P^V **- Speed handling qualities characteristics**

Уровень оценки	Центральная ручка	Штурвал
I	11	24
II	18	24
III	24	56

$\left(-\frac{\Delta P}{0,01M}\right)$

Уровень оценки	Центральная ручка	Штурвал
I	34	76.5
II	68	153
III	100	220

P, H

COMBINATION OF ALL AUTOMATIZATION IN THE SINGLE ON-BOARD SYSTEM

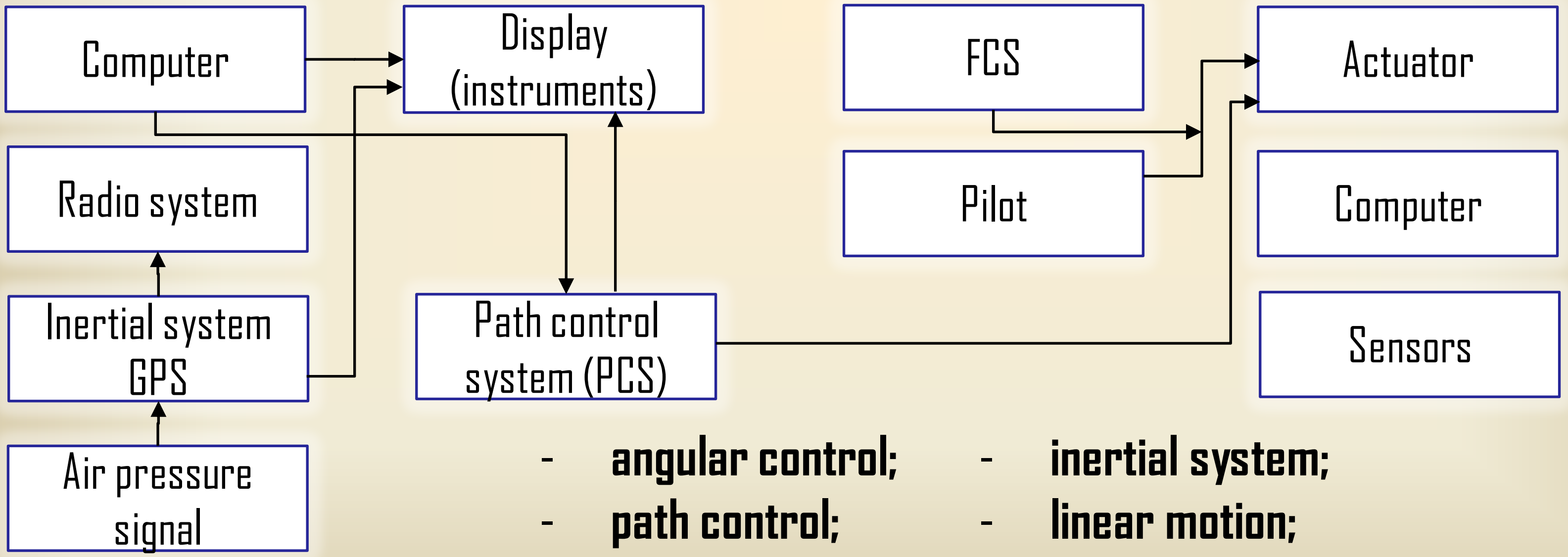
Tasks:

1. Improvement of flying qualities
2. Automatization of the initial flight regime
3. Autonatization of flight along the trajectory (path motion)
4. Limitation of critical regimes

FCS consists of the 3 main system

- Actuators
- Sensors
- On-board computer

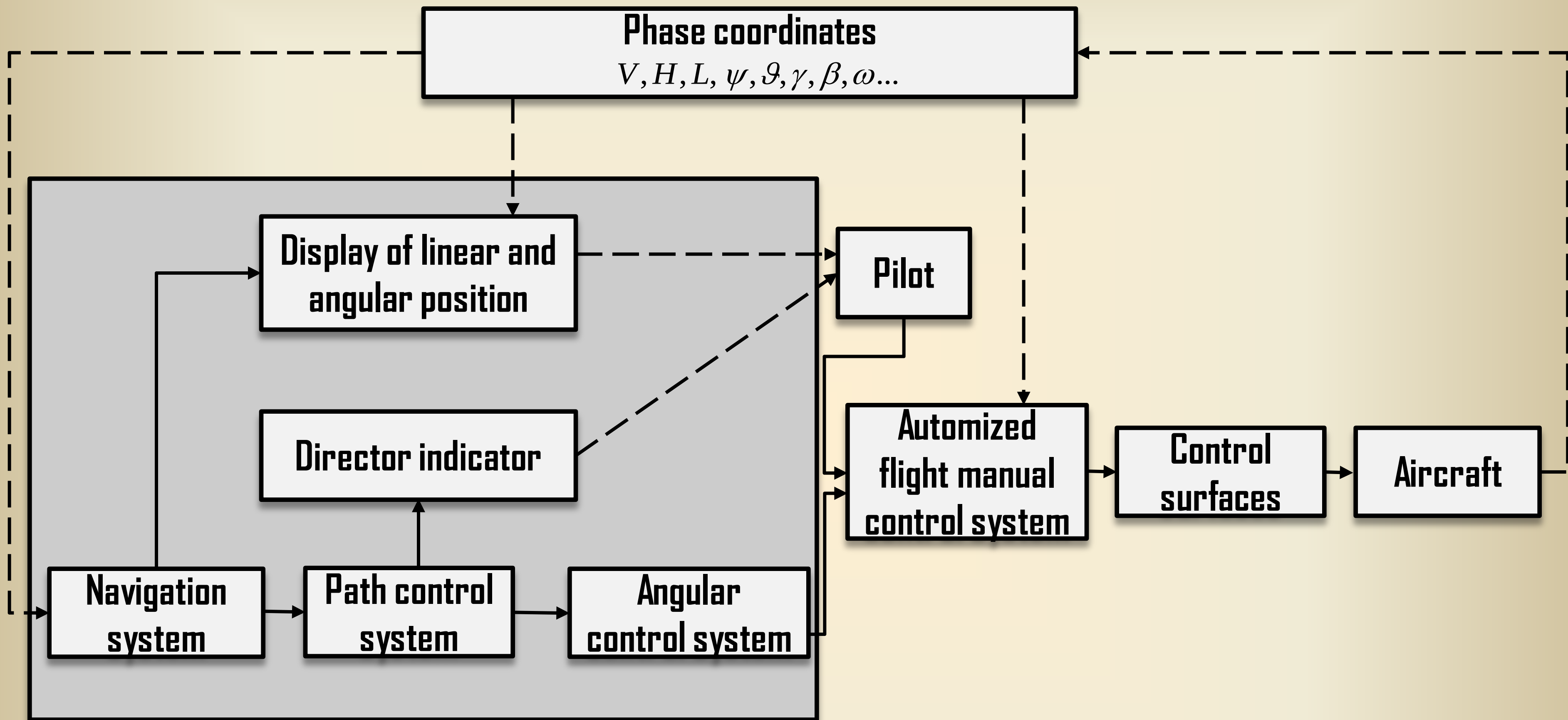
<u>Guidance-navigation system</u>



- angular control;
- path control;
- glide control;
- takeoff control

- inertial system;
- linear motion;
- V, ψ, ϑ, γ

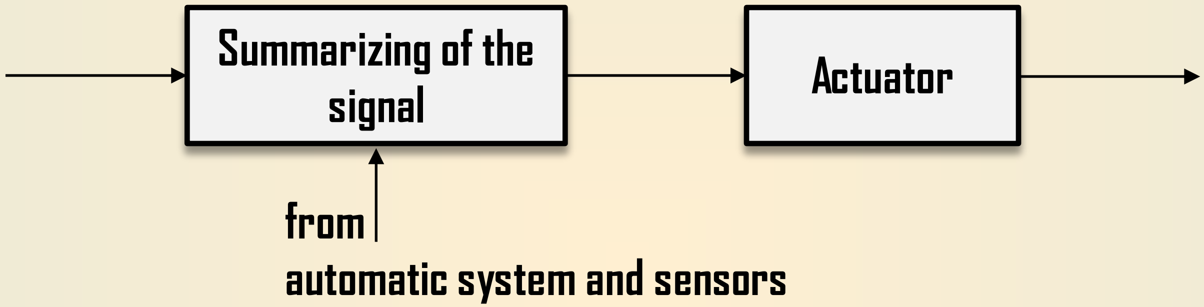
GUIDANCE AND NAVIGATION SYSTEM



FOUR TYPES OF CONTROL

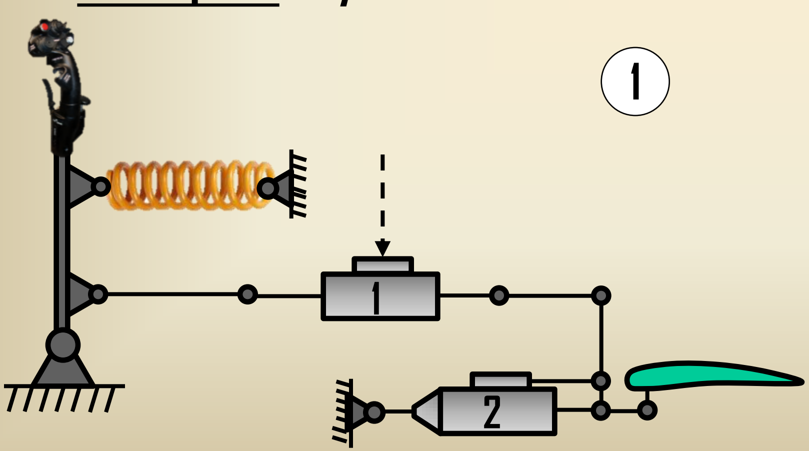
- Automatic path control
- Path control by use director indicator
- Automatic angular control
- Manual control with augmented flight control system

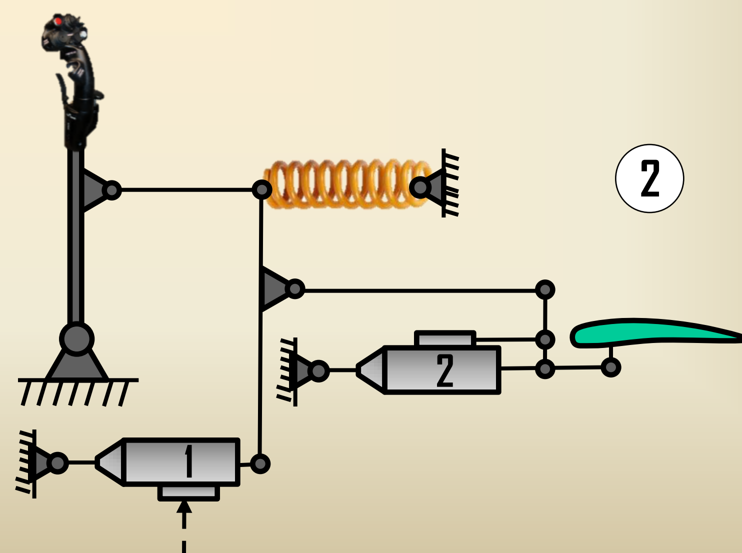
General scheme



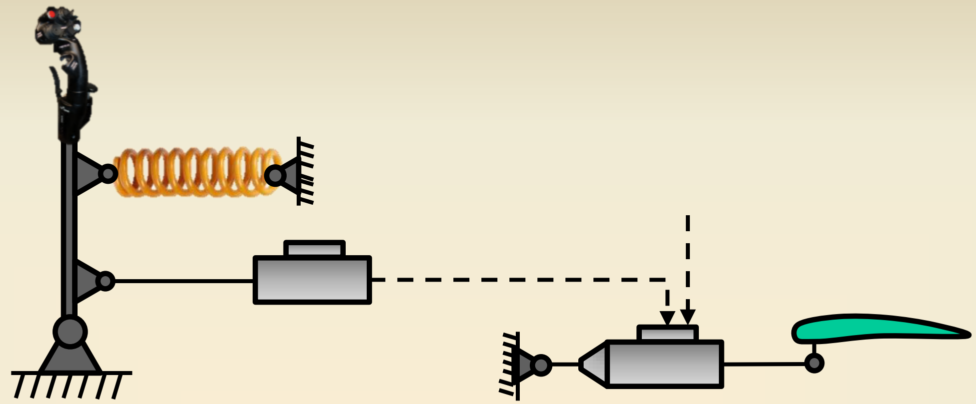
Ways of joining of actuator with pilot and sensor elements

In the past – by use of servo



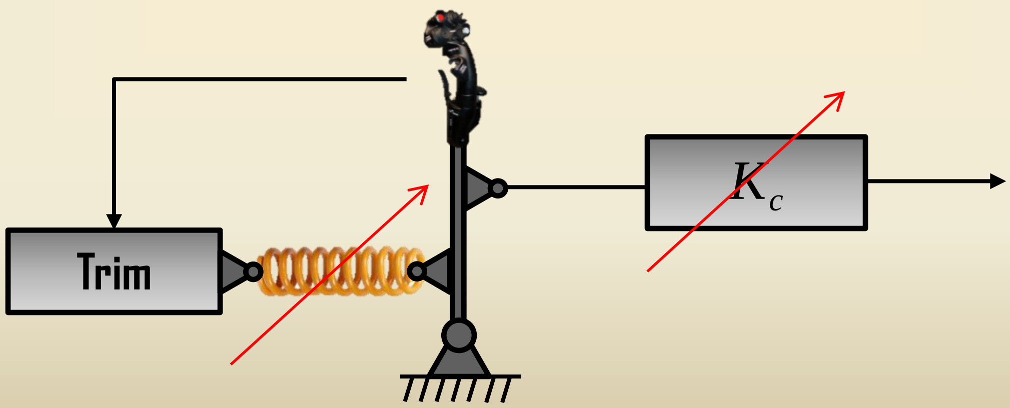


MODERN WAY: FLY-BY-WIRE



Additional elements spring with the regulation of its stiffness

- Mechanism for regulation of gain coefficient;
- Trimming mechanism.

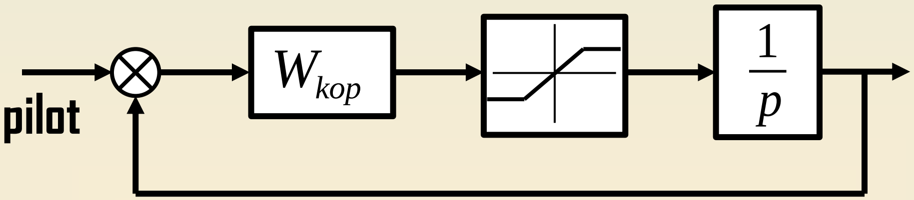


FLIGHT SAFETY – IS THE MAJOR REQUIREMENT TO THE FLIGHT CONTROL SYSTEM DESIGN

- Provision of the best flying qualities for all piloting tasks;
- Provision of conditions for suppression of possible nonlinear effects;
- Redundancy
- Limitation of critical regimes

MATHEMATICAL MODELS OF FCS ELEMENTS

ACTUATOR



Simple case $W_{kop} = K$

Linear model $W = \dfrac{1}{T_a p + 1}$

Requirements:

1) $\dfrac{1}{T_a} > 50 \div \omega_{sp}$, ω_{sp} - short period frequency

2) $\dot{\delta}_{\max} > 30\ {}^{0}/\,\mathrm{s}$ for civilian aircraft

$\dot{\delta}_{\max} > 60\ {}^{0}/\,\mathrm{s}$ for military aircraft

3) Power of actuator $P_a > 2P_h$

P_h - power necessary to suppress the high moment

SENSORS

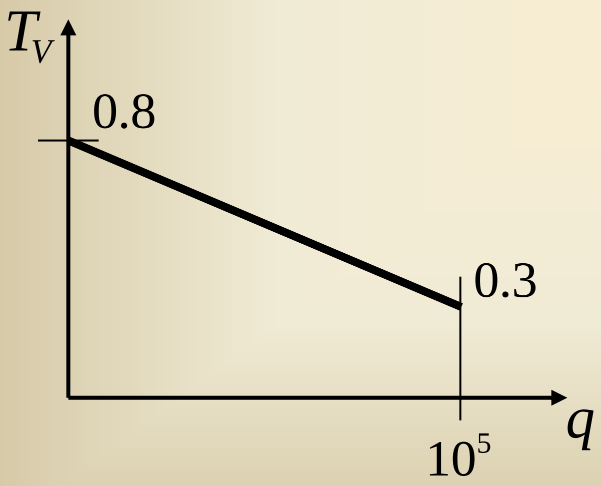
- Pitch rate sensor $W = \dfrac{1}{T^2 p^2 + 2\xi T p + 1}$, $\dfrac{1}{T} = 100 \;\; 1/c$, $\xi = 0.7$

- Accelerometer $W = \dfrac{1}{T_a^2 p^2 + 2\xi_a T_a p + 1}$, $\dfrac{1}{T_a} = 200 \;\; 1/c$, $\xi = 0.7$

The same dynamics of sensors α and β - the same

- Hydroscope sensors $W = 1$

INDICATE VELOCITY

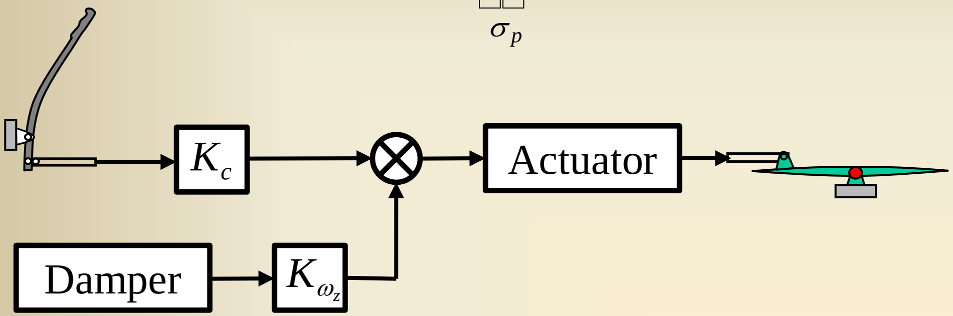


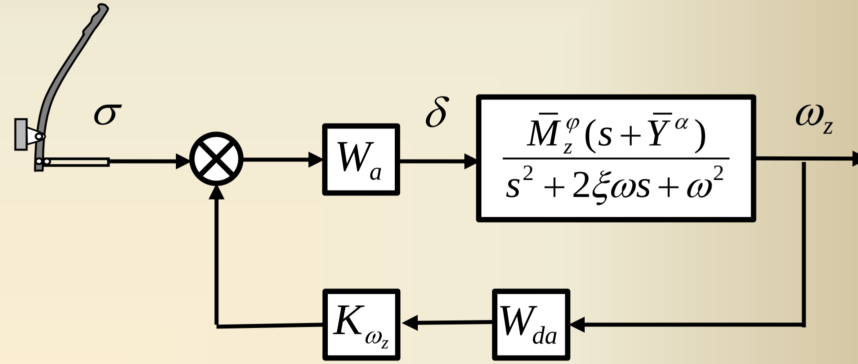
$$V_{ind} = V\sqrt{\Delta} \qquad \Delta = \frac{\rho_H}{\rho}$$

$$W = \frac{1}{T_V p + 1} \qquad T_V = 0.3 \div 0.8$$

Pitch rate damper

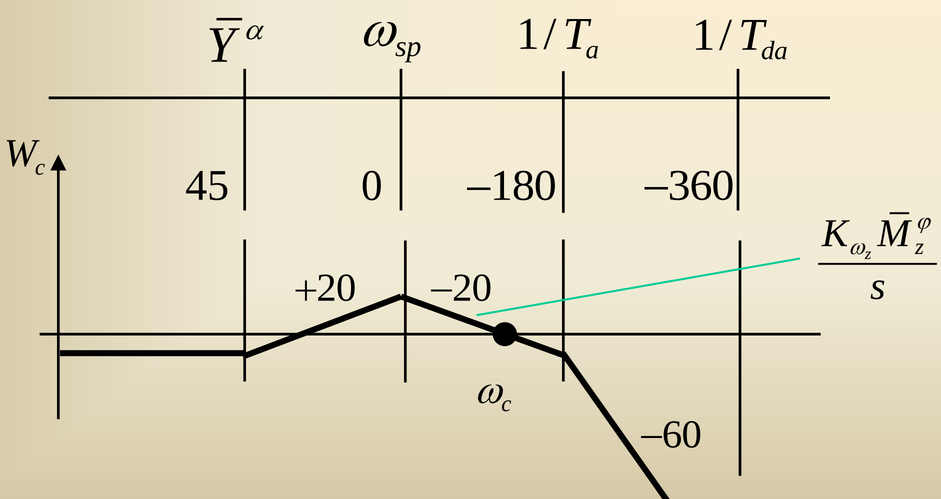
$$\sigma = \underbrace{X_c \cdot K_c}_{\sigma_p} + K_{\omega_z}\omega_z$$





$$\bar{Y}^{\alpha} < \omega_0 < \frac{1}{T_a} \qquad \frac{1}{T_a} < \frac{1}{T_{da}}$$

$$W_{\omega_z} = \frac{-K_{\omega_z}\bar{M}_z^{\varphi}(s+\bar{Y}^{\alpha})}{(s^2+2\xi_{sp}\omega_{sp}s+\omega_{sp}^2)(T_a^2s^2+2\xi_a\omega_a s+\omega_a^2)(T_{da}^2s^2+2\xi_{da}\omega_{da}s+\omega_{da}^2)}$$



$$\omega_c = -K_{\omega_z}\bar{M}_z^{\varphi}$$

$$\omega_{c\max} = -K_{\omega_z}^{\max}\bar{M}_z^{\varphi} = \frac{1}{T_a}$$

$$K_{\omega_z}^{\max} = \frac{1}{-\bar{M}_z^{\varphi}T_a} \rightarrow K_{\omega_z}^{*} = \frac{-0.25}{\bar{M}_z^{\varphi}T_a}$$

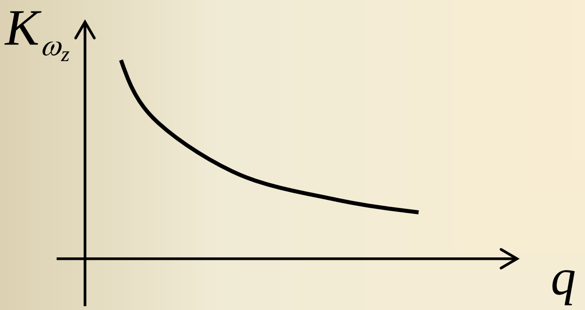
$$\Phi_{CL} = \frac{W_{OL}}{1+W_{OL}}\bigg|_{W_a = W_{da} = 1} = \frac{M_z^{\varphi}(s+\bar{Y}^{\alpha})}{s^2(2\xi\omega - K_{\omega_z}\bar{M}_z^{\varphi})s + \omega_k^2 - \bar{M}_z^{\varphi}K_{\omega_z}\bar{M}_z^{\varphi}\bar{Y}^{\alpha}}$$

$$2\xi\omega^* = 2\xi\omega - K_{\omega_z}\bar{M}_z^{\varphi}$$

$$\omega^{*2} = \omega_k^2 - K_{\omega_z}\bar{M}_z^{\varphi}\bar{Y}^{\alpha} \qquad \xi^* = \frac{2\xi\omega - 0.5K_{\omega_z}\bar{M}_z^{\varphi}}{\sqrt{\omega_k^2 - K_{\omega_z}\bar{M}_z^{\varphi}\bar{Y}^{\alpha}}}$$

$$If \quad \xi^* = \xi_{req} \quad \rightarrow \; K_{\omega_z};$$

$$K_{\omega_z} = \min\left\{K_{\omega_z *},\; K_{\omega_z\; req}\right\}$$



Influence of K_{ω_z} on static characteristics

$$\sigma_n = m_z^{C_y} - \frac{m_z^{\bar{\omega}_z}}{\mu} \qquad \frac{1}{\mu} = \frac{pSb_a}{2m}$$

$$\Delta m_z^{\bar{\omega}_z} = \bar{M}_z^{\varphi}K_{\omega_z}\frac{V}{b_a}$$

$$\Delta\sigma_n = \frac{g}{v}K_{\omega_z}\frac{m_z^{\varphi}}{C_{y\_hf}} < 0 \;\rightarrow \text{static stability increases} \rightarrow \omega_k \uparrow$$

$$\frac{\Delta X}{\Delta n_y} = \frac{\omega^2}{\bar{M}_z^{\varphi} n_y^{\alpha}} = \frac{\omega^2 - \bar{Y}^{\alpha} \bar{M}_z^{\varphi} K_{\omega_z}}{\bar{M}_z^{\varphi} n_y^{\alpha}} = X^{n_y} - K_{\omega_z} \frac{g}{V}$$

Example: IL-86

$H = 5 \qquad M = 0.78$

$\bar{Y}^{\alpha} = \frac{g}{V} n_y^{\alpha} = 0.865 \ [1/s]$ $\qquad \omega_k^2 = 2.62 \ [1/s^2]$ $\qquad \bar{M}_z^{\varphi} = -2.28$

$\omega_k = 1.62 \ [1/s]$ $\qquad 2\xi\omega = 1.684$ $\qquad \omega_a = 10\omega_k = 16.2 \ [1/s]$

$T_a = \frac{1}{\omega} = 0.05 \ [s]$ $\qquad K_{\omega_z}^* = \frac{0.25}{\bar{M}_z^{\varphi} T_a} = 2.2$ $\qquad \omega_a = 20; \ 25; \ 33 \ [1/s]$

K_{ω_z}	0	0.5	1.0	1.5	2.0	2.2
$2\xi\omega$	1.684	2.824	3.964	5.1	6.224	6.684
ω^*	1.62	1.9	2.14	2.36	2.56	2.64
ξ^*	0.52	0.743	0.426	1.08	1.22	1.27

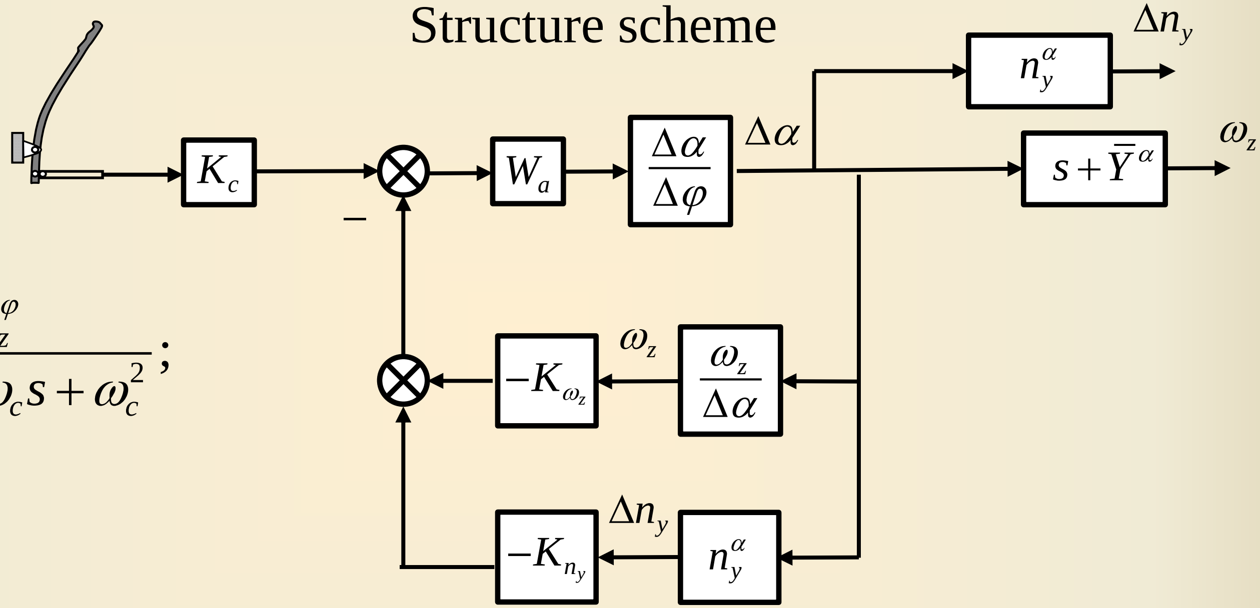
FCS with ω_z and n_y feedbacks

$$\Delta\varphi_a = K_{\omega_z}\omega_z + K_{n_y}\Delta n_y$$

Total signal to elevator

$$\varphi = K_c X_в + K_{\omega_z}\omega_z + K_{n_y}\Delta n_y$$

Structure scheme



$$\frac{\Delta\alpha}{\Delta\varphi} = \frac{M_z^\varphi}{s^2 + 2\xi_c\omega_c s + \omega_c^2};$$

$$\frac{\omega_z}{\Delta\alpha} = p + \bar{Y}^\alpha$$

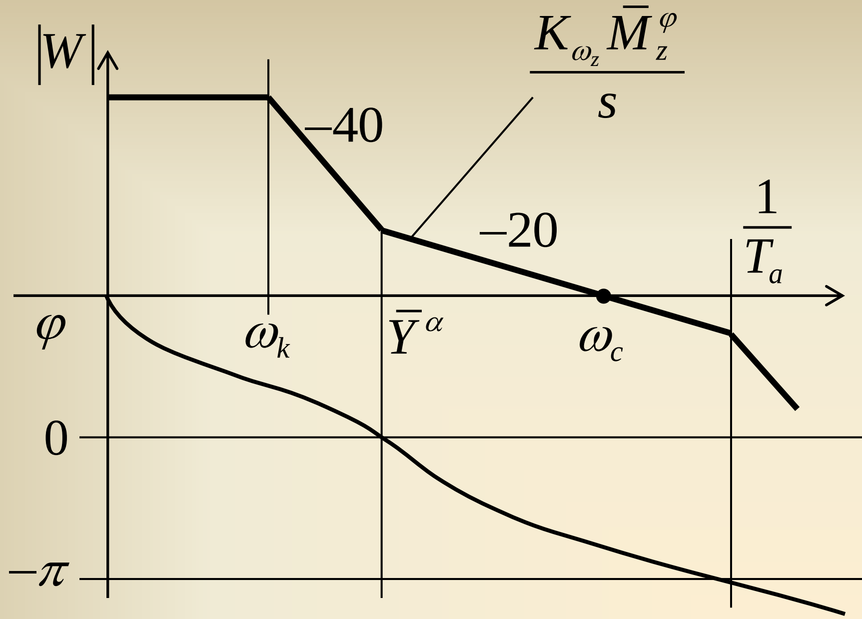
$$W_a = \frac{1}{T_a^2 s^2 + 2\xi_n\omega_n s + 1};$$

$$W_{f\beta} = -K_{\omega_z}(s+\bar{Y}^\alpha) - K_{n_y}\bar{Y}^\alpha = -K_{\omega_z}(s+\bar{Y}_*^\alpha);$$

$$Y^\alpha = \bar{Y}^\alpha + \lambda n_y^\alpha;$$

$$\lambda = \frac{K_{n_y}}{K_{\omega_z}};$$

$$W_{OL}^{\Delta\alpha} = \frac{K_{\omega_z}\left|M_z^\varphi\right|(s+\bar{Y}_*^\alpha)}{(\mathrm{s}^2 + 2\xi_k\omega_k\,\mathrm{s} + \omega_k^2)(\mathrm{T}_a^2\mathrm{s}^2 + 2\,\mathrm{T}_a\,\xi_a + 1)};$$



$$\omega_c = K_{\omega_z}\bar{M}_z^{\varphi} \to K_{\omega_z \max} = \frac{1}{T_a \bar{M}_z^{\varphi}}$$

$\varepsilon = \frac{K_{\omega_z}}{K_{\omega_z \max}}$ - defines amplitude margin

$\bar{Y}_*^{\alpha} \cdot T_a$ - characterizes phase margin

(decrease of $\bar{Y}_*^{\alpha} \cdot T_a$ - increase of phase margin)

$$\varepsilon \le 0.15; \qquad \bar{Y}_*^{\alpha} \le \frac{0.2}{T_a}$$

Let's define the desired values

$$2\xi_{eff}\,\omega_{eff}$$

$$\frac{\Delta\alpha}{\Delta X_{в}} = \frac{K_{ш}\bar{M}_z^{\varphi}}{\Delta eff\,(s)} \qquad \Delta eff\,(s) = s^2 + 2\,(\xi_k\omega_k - 0.5K_{\omega}\bar{M}_z^{\varphi})\,s + \omega_k^2 + \bar{M}_z^{\varphi}\bar{Y}_*^{\alpha}$$

$$\omega_{eff}^2$$

$$\omega_{eff}^2 = \omega_k^2 + \omega_c\bar{Y}_*^{\alpha} \qquad \xi_{c\ eff} = \frac{\omega_0\xi_k + 0.5\omega_c}{\sqrt{\omega_k^2 + \omega_c} \cdot \bar{Y}_*^{\alpha}}$$

$$\omega_c = -K_{\omega_z} \bar{M}_z^{\varphi};\qquad \bar{Y}_*^{\alpha} = \bar{Y}^{\alpha} + \Delta n_y^{\alpha}$$

ω_k^* and ξ_k^* desired values of $\omega_{c\,eff}$ and $\xi_{c\,eff}$

Then

$$\omega_c = \frac{\omega_k^{*2} - \omega_k^2}{\bar{Y}_*^{\alpha}}$$

$$\bar{Y}_*^{\alpha} = \frac{(\omega_k \xi_k + 0.5\omega_k)^2 \dfrac{2}{(\xi^*)^2} - \omega_k^2}{\omega_c}$$

$$\begin{cases} \omega_c^{req} = 2(\xi_* \omega^* - \xi_k \omega_k) \\ \bar{Y}_{*\mathrm{req}}^{\alpha} = \dfrac{\omega^{*2} - \omega_k^2}{\omega_c^{req}} \end{cases}$$

$$\bar{Y}_{*\mathrm{req}}^{\alpha} \le \frac{0.2}{T_{a\,req}}\qquad K_{\omega_z}^{req} \le \frac{\omega_c}{\bar{M}_z^{\varphi}}$$

$$\lambda^0 = \frac{1}{n_y^{\alpha}} (\bar{Y}_{*\mathrm{req}}^{\alpha} - \bar{Y}^{\alpha})$$

Rational values

$$K_{\omega_z} = \min\left\{K_{\omega_z}^{req};\ 0.25 K_{\omega_z\,\max}\right\}$$

$$K_{n_y} = \lambda^0 K_{\omega_z}$$

After it is necessary to define $K_{\omega_z}^*(q)$; $K_{n_y} = A$

After regulation

$$\omega_{k\,eff} = \sqrt{\omega_k^2 - \bar{M}_z^{\varphi} n_y^{\alpha} (\frac{g}{V} K^*_{\omega_z} + K^*_{n_y})}$$

$$\xi_{k\,eff} = \frac{\xi_k \omega_k - 0.5 K^*_{\omega_z} \bar{M}_z^{\varphi}}{\omega_{0\,eff}}$$

Let's define the influence of feedback on X^{n_y} and σ_n

$$\Delta\sigma_n = \Delta m_z^{C_y} + \frac{\Delta m_z^{\bar{\omega}_z}}{\mu}; \qquad \Delta m_z^{\bar{\omega}_z} = \frac{K_{\omega_z} \bar{M}_z^{\xi} V}{b_a}; \qquad \Delta m_z^{C_y} = K_{n_y} \bar{M}_z^{\varphi} \frac{qS}{mg};$$

$$\Delta\sigma_n = \frac{m_z^{\varphi} qS}{mg} (K_{n_y} + K_{\omega z} \frac{g}{V}); \qquad \frac{1}{\mu} = \frac{\rho S b_a}{2m};$$

$$\frac{\Delta m_z^{\omega_z}}{\mu} = \frac{K_{\omega_z} m_z^{\varphi}}{b_a} \frac{\rho S b_a V g V}{2mgV} = \frac{\rho V^2}{2} \frac{m_z^{\varphi}}{mg} [K_{\omega_z} \frac{g}{V}];$$

Thus the feedback lead to increase of the <u>stability</u>.

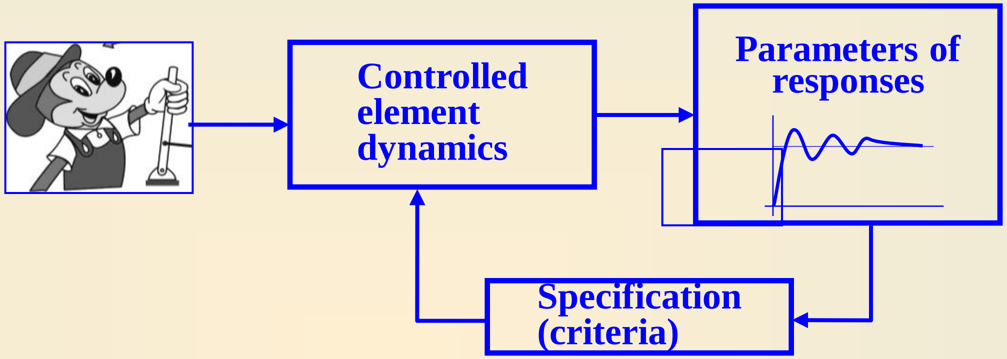
What are the flying qualities

«Flying qualities of an aircraft are those properties which describe the ease and effectiveness with which it responds to pilot commands in the execution of some flight task».

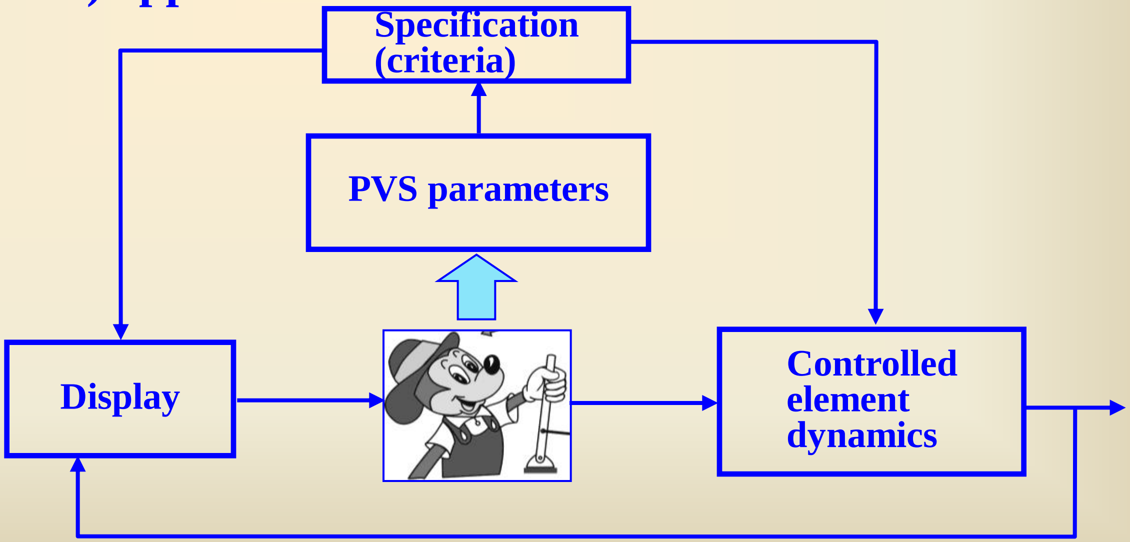
D. McRuer
M. Cock

How to select the flying qualities?

1. Traditional way



2. Pilot–vehicle system (PVS) approach



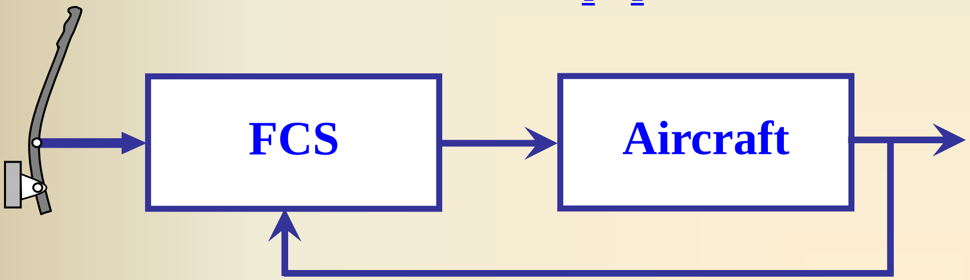
Criteria – are the requirements to the FQ

CRITERIA:

- **effectiveness in fulfillment of piloting tasks (accuracy)**
- **flight safety**

Criteria – used now for flight control system design

a. Effectiveness is provided by flying qualities corresponding to the specific boundary of Aircraft + Flight Control System parameters f(a_1, a_2, …)

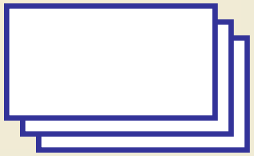


Flying qualities level

в. Flight safety is provided by fixed reliability of aircraft subsystem

probability of accident for passenger airplanes $p = 10^{-9}$

aircraft subsystems



Some regularities of pilot behavior

Adaptation of human behavior

“A mathematical investigation of controlled motion is rendered almost impossible on account of the adaptability of the pilot”
W. Crawley (1930)

Main task variables influenced on adaptation:
– controlled element dynamics, input signal

Open loop “crossover model”

$$W_p W_C = \frac{\omega_C}{j\omega} e^{-j\omega\tau_e}$$

$$\omega_c = f(W_C, S_{ii}) + \Delta\omega_c(\omega_i)$$

$$\tau_e = f(W_C, S_{ii}(\omega))$$

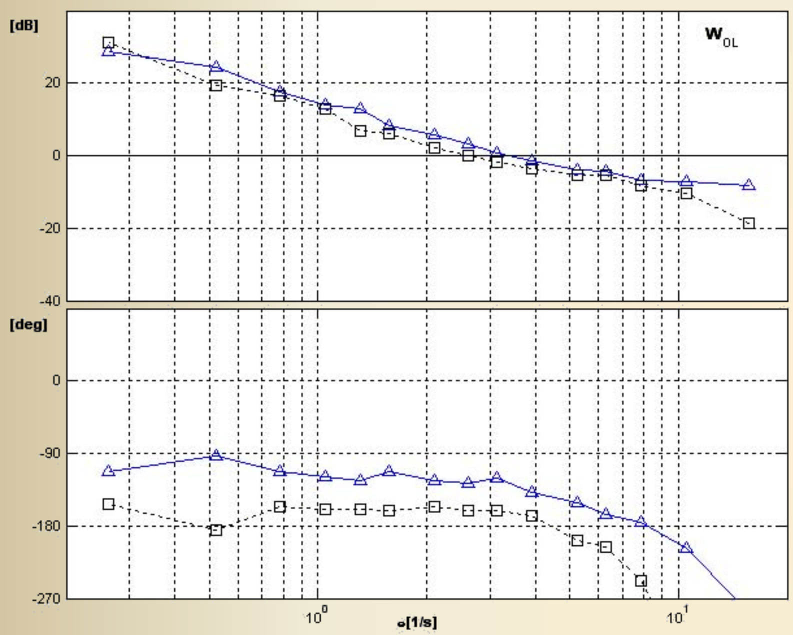
Crossover pilot model

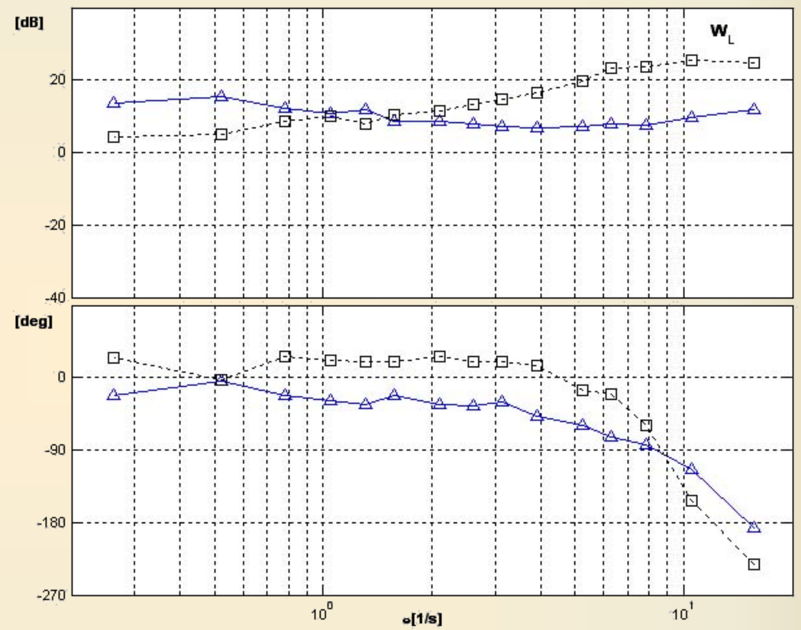
$$W_p\Big|_{\omega_c} = K_p \frac{T_L j\omega + 1}{T_I j\omega + 1} e^{-j\omega\tau}$$

$$W_c = \frac{K_c}{j\omega} \Rightarrow W_p = K_p e^{-j\omega\tau}$$

$$W_c = \frac{K_c}{j\omega(Tj\omega + 1)} \Rightarrow W_p = K_p (T_L j\omega + 1) e^{-j\omega\tau}$$

Example: Experimental investigation of pilot adaptation





□ – $\frac{K_0}{s^2}$

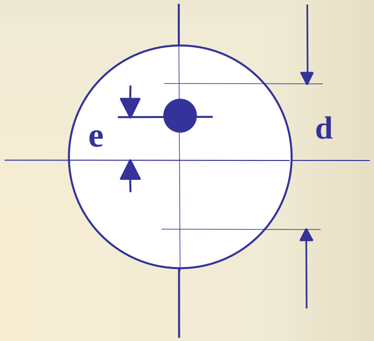
△ – $\frac{K_0}{s}$

$$W_C : \frac{K}{s} \Rightarrow \frac{K}{s^2} \Rightarrow \omega_C \downarrow;\ \tau_e \ ;\ \left.\frac{dW_p}{d\omega}\right|_{\omega_C} \quad \sigma_e^2 \uparrow;\ PR \uparrow$$

Considerable influence of task performance parameter

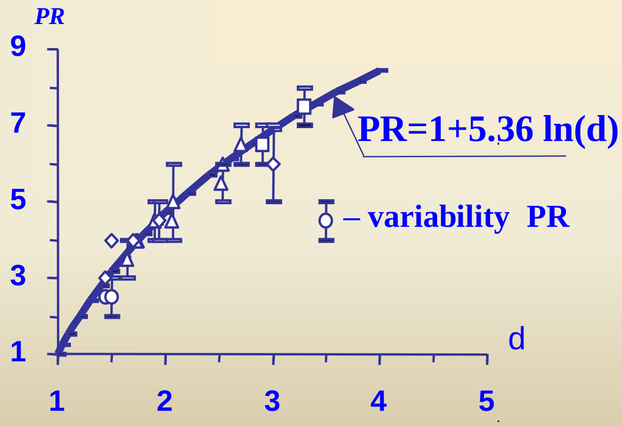
Table from WL-TR-96-3109

d [sm]	0.5	1.0	1.5	2.0
r [dB]	8.15	7.53	6.3	2.3
$\Delta\varphi_p$ [deg]	45	40	27	12
PR	8.5	8.0	6.0	3.5

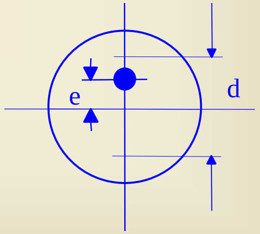


Agreement between Cooper–Harper pilot rating (*PR*) and Weber–Fechner law

Stimulas, (s) → **operator** → **Response, (R)** ⟹ $R = a + b \cdot \ln S$



Data base:
1. Neal Smith
2. Have PIO
3. LAHOS



Influence of the regulator on the static handling qualities

$$\frac{\Delta n_y}{\Delta X} = \frac{n_y^{\alpha} M_z^{\varphi} K_c}{\omega_{k\,eff}^2} \qquad X^{n_y} = \frac{\omega_{k\,eff}^2}{n_y^{\alpha} M_z^{\varphi} K_c} = \frac{\varphi^{n_y}}{K_c} - \underbrace{\frac{1}{K_c}\left(K_{\omega_z}\frac{g}{V} + K_{n_y}\right)}_{\text{influence of automatization}}$$

$$\left|X^{n_y}\right| = \frac{1}{K_c}\Big[\underbrace{\left|\frac{\varphi^{n_y}}{K_c}\right| + \left(K_{\omega_z}\frac{g}{V} + K_{n_y}\right)}_{F(q,H)}\Big] = c$$

$$\varphi^{n_y} = \frac{mg}{Sm_z^{\varphi}}\sigma_n \qquad K_c = \frac{F(q,H)}{c}$$

FCS with integral law and ω_z, n_y feedbacks (integral regulators)

$$\varphi = K_c X_в + K_{\omega_z}\omega_z + K_{n_y} n_y + K_{\int}\int(\Delta n_y + K_X \Delta X dt)$$

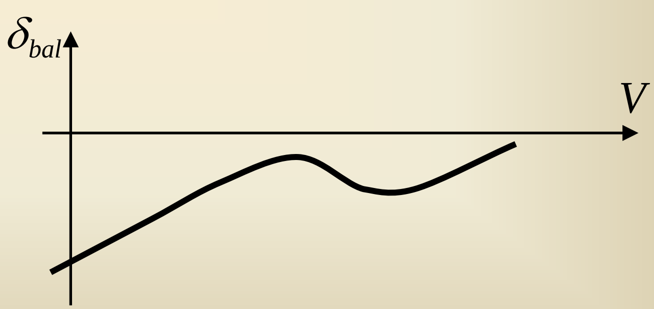
$$\Delta n_y = n_y - 1 \qquad \Delta X = X_в - X_{bal}$$

desired - balance curve

The specific peculiarities of such regulator

1. Provision of the constant X^{n_y}

2. Suppression of the "balance trim curve" peculiarity



For the steady flight: $\varphi = const;$ $X_в = const;$

$\omega_z = const;$ $\Delta n_y = const;$ $\Delta X = const;$

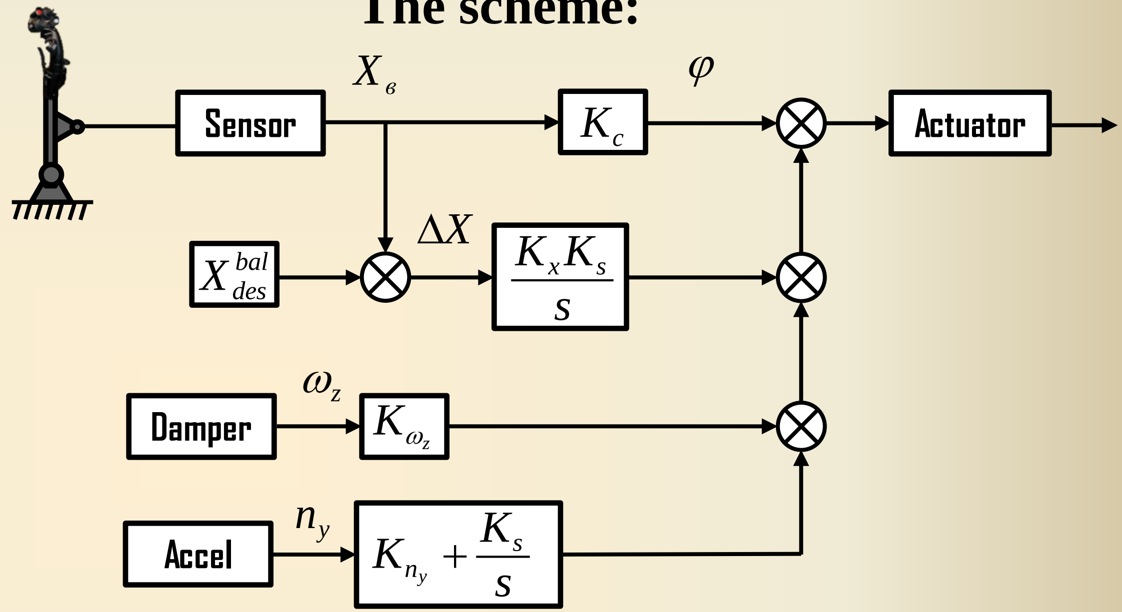
For the steady flight integral has to be equal zero

$$\Delta n_y + K_x \Delta X = 0$$

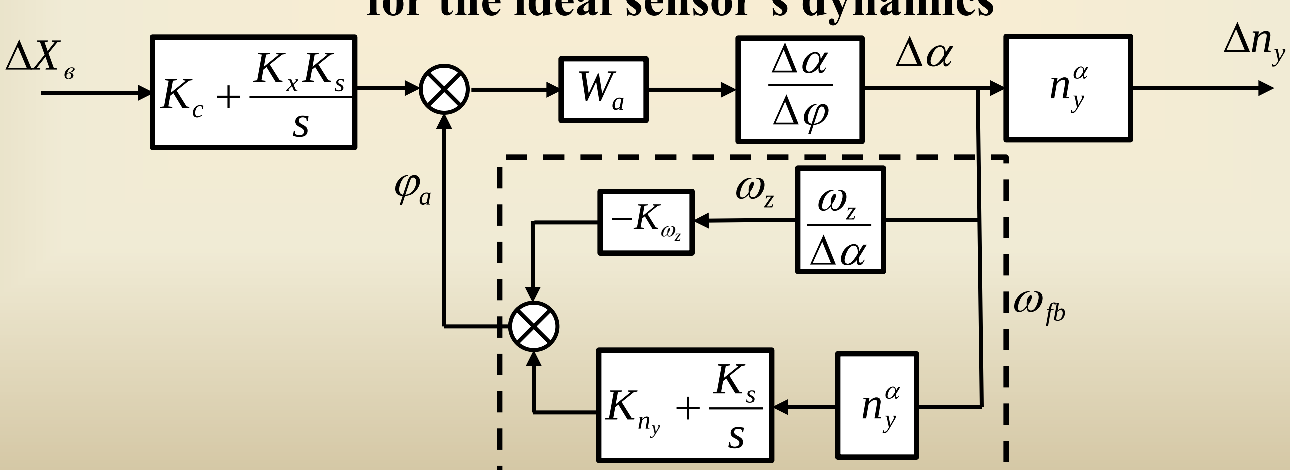
$$\Downarrow$$

$$X^{n_y} = \left.\frac{\Delta X}{\Delta n_y}\right|_{steady} = -\frac{1}{K_x}$$

The scheme:



for the ideal sensor's dynamics



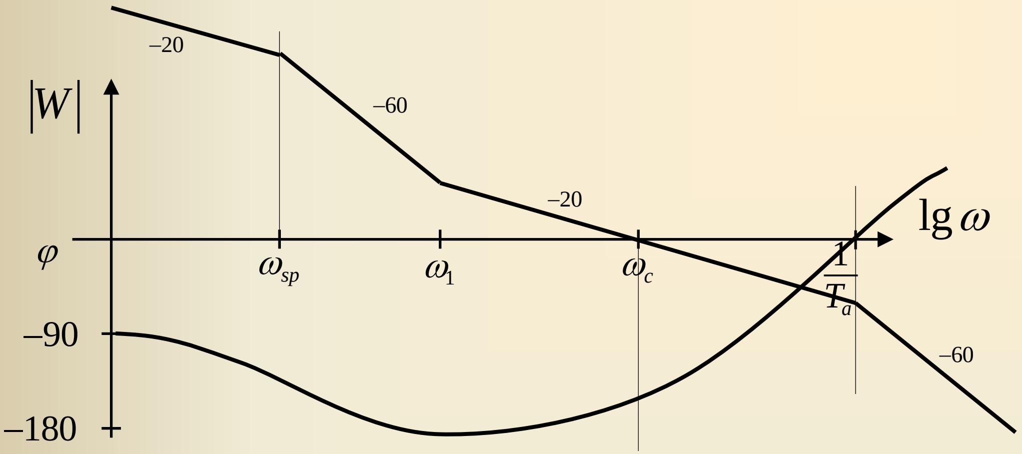
because of $$\frac{\Delta\alpha}{\Delta\varphi} = \frac{M_z^{\varphi}}{s^2 + 2\xi\omega_{sp}s + \omega_{sp}^2}$$

$$\frac{\omega_z}{\Delta\alpha} = s + \bar{Y}^{\alpha}$$

$$W_{fb} = -K_{\omega_z}\frac{s^2 + (\bar{Y}^{\alpha} + \frac{K_{n_y}}{K_{\omega_z}}n_y^{\alpha})s + \frac{K_{\int}}{K_{\omega_z}}n_y^{\alpha}}{s} = -K_{\omega_z}\frac{s^2 + 2h_1s + \omega_1^2}{s}$$

In case when $$W_a = \frac{1}{T_a^2s^2 + 2\xi_aT_as + 1}$$

$$W_{OL}^{\Delta\alpha} = \frac{K_{\omega_z}\left|M_z^{\varphi}\right|(s^2 + 2h_1s + \omega_1^2)}{s(s^2 + 2\xi\omega_{sp}s + \omega_{sp}^2)(T_a^2s^2 + 2\xi_a\omega_as + 1)}$$



$$\omega_c = K_{\omega_z}M_z^{\varphi}$$

$$\omega_c^{\max} = \frac{1}{T_a}$$

$$K_{\omega_z\max} = \frac{1}{T_a\left|M_z^{\varphi}\right|}$$

Requirements to amplitude and phase margins

- Amplitude margin $$\varepsilon = \frac{K_{\omega_z}}{K_{\omega_z\max}} \le 0.4$$

- Phase margin $$\max(\omega_{sp},\ \omega_1) \le \frac{0.15}{T_a}$$

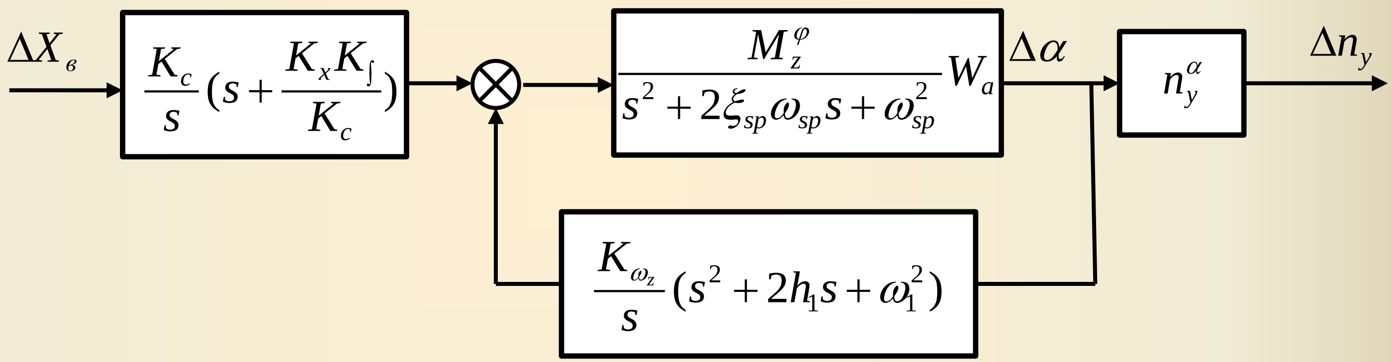
Let's select $\varepsilon^0 = 0.4$; $\omega_1^0 = (0.1 \div 0.15)\dfrac{1}{T_a}$; $\xi_1^0 = 0.8$; $\xi = \dfrac{h_1}{2\omega_1}$;

In that case $K_{\omega_z}^0 = \varepsilon^0 \cdot K_{\omega_z \max}$ $K_{n_y} = (2h_1 - \bar{Y}^{\alpha})\dfrac{1}{n_y^{\alpha}} K_{\omega_z}^0$ $K_{\int}^0 = \dfrac{\omega_1^{02}}{n_y^{\alpha}} K_{\omega_z}^0$

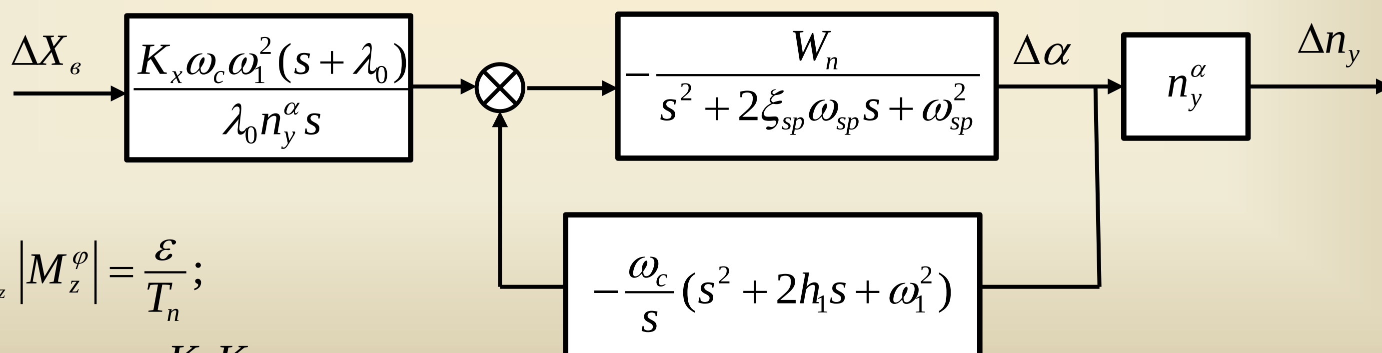
Procedure for selection of coefficients K_x **and** K_c

Let's define the closed-loop system

The scheme of initial open-loop system is the following



Transform it to the following



where $\omega_c = K_{\omega_z}\left|M_z^{\varphi}\right| = \dfrac{\varepsilon}{T_n}$;

$\lambda_0 = \dfrac{K_x K_{\int}}{K_c}$ **and** $K_c = \dfrac{K_x K_{\int}}{\lambda_0}$

In case actuator with the ideal dynamics:

$$\frac{\Delta n_y}{\Delta X} = -\frac{K_x \omega_c \omega_1^2 (s+\lambda_0)\frac{1}{\lambda_0}}{s(s^2+2\xi_{sp}\omega_{sp}s+\omega_{sp}^2)+\omega_c(s^2+2h_1 s+\omega_1^2)} \qquad (*)$$

From here $\left.\frac{\Delta n_y}{\Delta X}\right|_{\substack{steary \\ t\to\infty}} = \left.\frac{\Delta n_y}{\Delta X}\right|_{s=0} = -K_x$

We can select $K_x = \left|\frac{1}{X_b^{n_y}}\right|_{desired}$

λ_0 **is the reason of the oscillation of the close-loop system. Its selection can be carried out by the following way:**

The denominator of the (*) $\Rightarrow$

$$\Rightarrow \quad s^3+(2\xi_{sp}\omega_{sp}+\omega_c)s^2+(2h_1\omega_c+\omega_{sp}^2)s+\omega_c\omega_1^2 = (s^2+2h_\Delta s+\omega_\Delta^2)(s+\lambda_1)$$

$$\omega_\Delta \approx \omega_1; \quad h_\Delta \approx h_1; \quad \Rightarrow \quad \lambda_1 = \omega_c = \frac{\varepsilon^0}{T_n}$$

Select $\lambda_0 = \lambda_1 \quad \Rightarrow \quad \left(\frac{K_c}{K_x}\right)^0 = \frac{K_\int^0}{\omega_c} = \frac{K_\int^0}{\varepsilon^0} T_n$

This requirement will suppress the effect of λ_0

FCS with the feedbacks n_y and ω_z for statically unstable aircraft

For statically unstable aircraft

$$\omega^2 = -\frac{qSb_a}{I_z}\sigma_n < 0 \qquad \Delta = s^2 + 2\xi\omega_{sp}s + \omega_{sp}^2 = (s+\lambda_1)(s-\lambda_2)$$

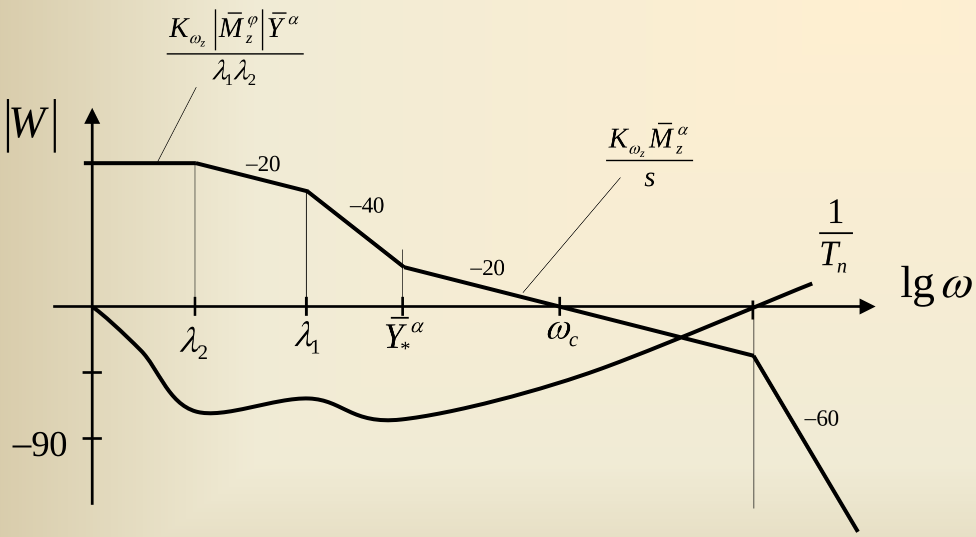
$$\lambda_1 > \lambda_2$$

$$X^{n_y} = \frac{1}{K_{ш}}\left[\underbrace{-\frac{\sigma_n m \frac{g}{S}}{m_z^{\varphi} q}}_{"+"} \underbrace{- K_{\omega_z}\left(\frac{g}{V} + \frac{K_{n_y}}{K_{\omega_z}}\right)}_{"*"}\right]$$

The necessary sign = (-) is provided by (*)

In that case the open-loop system

$$W_{OL}^{\Delta\alpha} = \frac{K_{\omega_z}\left|\bar{M}_z^{\varphi}\right|(s+\bar{Y}_*^{\alpha})}{(s+\lambda_1)(s-\lambda_2)(T_a^2 s^2 + 2\xi_a T_a s + 1)}$$



$$\bar{Y}_*^{\alpha} = \bar{Y}^{\alpha} + \frac{K_n}{K_{\omega_z}} n_y^{\alpha}$$

$$K_{\omega_z \max}^{+} = \frac{1}{T_a\left|\bar{M}_z^{\varphi}\right|}$$

$$\frac{K_{\omega_z}\left|\bar{M}_z^{\varphi}\right|\bar{Y}_*^{\alpha}}{\lambda_1\lambda_2} = 1$$

$$\Downarrow$$

$$K_{\omega_z \max}^{-} = \frac{\lambda_1\lambda_2}{\left|\bar{M}_z^{\varphi}\right|\left(\frac{g}{V} + \frac{K_{n_y}}{K_{\omega_z}}\right)n_y^{\alpha}}$$

The provision of necessary phase margin

$$\bar{Y}_{*}^{\alpha} < \frac{0.15}{T_n} \Rightarrow \frac{K_{n_y}}{K_{\omega_z}} \le \frac{0.15}{T_a n_y^{\alpha}} - \frac{g}{V}$$

$$2.5K_{\omega_z \max}^{-} < K_{\omega_z} < 0.4K_{\omega_z \max}^{+}$$

THANK YOU FOR ATTENTION!